

# Ohio's State Tests

PRACTICE TEST ANSWER KEY & SCORING GUIDELINES

**GEOMETRY** 

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Question No.	Item Type	Content Domain	Content Standard	Answer Key	Points
1	Multiple Choice	Circles	Prove that all circles are similar. (G.C.1)	С	1 point
2	Graphic Response	Congruence	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. (G.CO.5)		1 point
3	Hot Text Item	Expressing Geometric Properties with Equations	Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point $(0, 2)$ . $(G.GPE.4)$		1 point
4	Graphic Response	Similarity, Right Triangles, and Trigonometry	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. (G.SRT.6)		1 point
5	Multi- Select Item	Conditional Probability and the Rules of Probability	Understand the conditional probability of A given B as P (A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. (S.CP.3)	A, D, F	1 point
6	Short Response	Circles	Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. (G.C.2)		1 point

Question	ltem	Item Content Content Answer Reinte				
No.	Type	Domain	Standard	Key	Points	
7	Hot Text Item	Congruence	Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. (G.CO.10)		1 point	
8	Multiple Choice	Congruence	Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. (G.CO.11)	В	1 point	
9	Multiple Choice	Congruence	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. (G.CO.12)	D	1 point	

Question No.	Item Type	Content Domain	Content Standard	Answer Key	Points
10	Multiple Choice	Congruence	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. (G.CO.3)	D	1 point
11	Multi- Select Item	Congruence	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. (G.CO.6)	C, D	1 point
12	Equation Item	Geometric Measurement and Dimension	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. (G.GMD.3)		1 point
13	Multiple Choice	Geometric Measurement and Dimension	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. (G.GMD.4)	В	1 point

Question No.	Item Type	Content Domain	Content Standard	Answer Key	Points
14	Equation Item	Expressing Geometric Properties with Equations	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. (G.GPE.1)		1 point
15	Equation Item	Expressing Geometric Properties with Equations	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). (G.GPE.5)		1 point
16	Equation Item	Expressing Geometric Properties with Equations	Find the point on a directed line segment between two given points that partitions the segment in a given ratio. (G.GPE.6)		1 point
17	Equation Item	Expressing Geometric Properties with Equations	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. (G.GPE.7)		1 point
18	Equation Item	Modeling with Geometry	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). (G.MG.3)		2 points

Question No.	Item Type	Content Domain	Content Standard	Answer Key	Points
19	Multiple Choice	Similarity, Right Triangles, and Trigonometry	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. (G.SRT.2)	Α	1 point
20	Equation Item	Similarity, Right Triangles, and Trigonometry	Verify experimentally the properties of dilations given by a center and a scale factor:  b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. (G.SRT.1b)		1 point
21	Hot Text Item	Similarity, Right Triangles, and Trigonometry	Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. (G.SRT.4)		1 point
22	Equation Item	Similarity, Right Triangles, and Trigonometry	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. (G.SRT.8)		1 point
23	Equation Item	Similarity, Right Triangles, and Trigonometry	Explain and use the relationship between the sine and cosine of complementary angles. (G.SRT.7)		1 point

Question No.	Item Type	Content Domain	Content Standard	Answer Key	Points
24	Equation Item	Conditional Probability and the Rules of Probability	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. (S.CP.2)		1 point
25	Table Item	Conditional Probability and the Rules of Probability	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among Math, Science, and English. Estimate the probability that a randomly selected student from your school will favor Science given that the student is in tenth grade. Do the same for other subjects and compare the results. (S.CP.4)		1 point

Question No.	Item Type	Content Domain	Content Standard	Answer Key	Points
26	Multiple Choice	Conditional Probability and the Rules of Probability	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer. (S.CP.5)	D	1 point
27	Equation Item	Conditional Probability and the Rules of Probability	Apply the Addition Rule, P(A or B) = P(A) + P(B) - P(A and B), and interpret the answer in terms of the model. (S.CP.7)		1 point
28	Equation Item	Congruence	Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). (G.CO.2)		1 point

Geometry
Practice Test

**Question 1** 

**Question and Scoring Guidelines** 

#### **Question 1**

Circle J is located in the first quadrant with center (a, b) and radius s. Felipe transforms Circle J to prove that it is similar to any circle centered at the origin with radius t.

Which sequence of transformations did Felipe use?

- Translate Circle J by (x + a, y + b) and dilate by a factor of  $\frac{t}{s}$ .
- Translate Circle J by (x + a, y + b) and dilate by a factor of  $\frac{s}{t}$ .
- Translate Circle J by (x-a, y-b) and dilate by a factor of  $\frac{t}{s}$ .
- Translate Circle J by (x-a, y-b) and dilate by a factor of  $\frac{s}{t}$ .

Points Possible: 1

**Content Domain: Circles** 

**Content Standard:** Prove that all circles are similar. (G.C.1)

### **Scoring Guidelines**

<u>Rationale for Option A:</u> This is incorrect. The student may have noticed that the Circle *J* is located to the right of Circle *O*, and may have thought that he or she needed to translate the center of Circle *O* to the right *a* units and up *b* units, and to use addition to represent this translation.

<u>Rationale for Option B:</u> This is incorrect. The student may have noticed that Circle J is located to the right of Circle O, and may have thought that he or she needed to translate the center of Circle O to the right a units and up b units and to use addition to represent this translation. The student may have also used the inverse of the correct scale factor.

Rationale for Option C: **Key** – The student recognized that translating Circle J with the center at (a, b) to the origin (0, 0) involves a subtraction of a units from the x-coordinate and b units from the y-coordinate of the center of Circle J. Dilation by a scale factor  $\frac{t}{s}$  (radius of the image/radius of the pre-image) overlays Circle J on any other circle centered at the origin with radius t, proving a similarity.

Rationale for Option D: This is incorrect. The student recognized that translating the Circle J with the center at (a, b) to the origin (0, 0) involves a subtraction of a units from the x-coordinate and b units from the y-coordinate of the center of Circle J, but he or she used the inverse of the correct scale factor.

#### Sample Response: 1 point

Circle J is located in the first quadrant with center (a, b) and radius s. Felipe transforms Circle J to prove that it is similar to any circle centered at the origin with radius t.

Which sequence of transformations did Felipe use?

- Translate Circle J by (x + a, y + b) and dilate by a factor of  $\frac{t}{s}$ .
- Translate Circle J by (x + a, y + b) and dilate by a factor of  $\frac{s}{t}$ .
- Translate Circle J by (x-a, y-b) and dilate by a factor of  $\frac{t}{s}$ .
- Translate Circle J by (x-a, y-b) and dilate by a factor of  $\frac{s}{t}$ .

Geometry
Practice Test

**Question 2** 

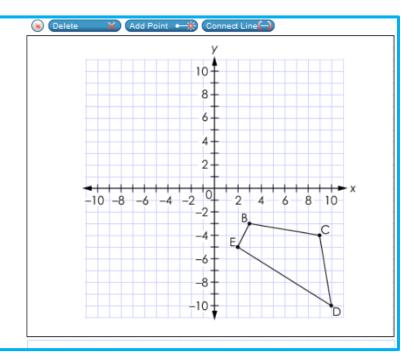
**Question and Scoring Guidelines** 

#### **Question 2**

Quadrilateral BCDE is shown on the coordinate grid.

Keisha reflects the figure across the line y = x to create B'C'D'E'.

Use the Connect Line tool to draw quadrilateral B'C'D'E'.



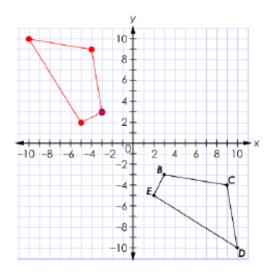
Points Possible: 1

Content Domain: Congruence

**Content Standard:** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. (G.CO.5)

### **Scoring Guidelines**

#### Exemplar Response



#### Other Correct Responses

• Additional lines and points are ignored.

For this item, a full-credit response includes:

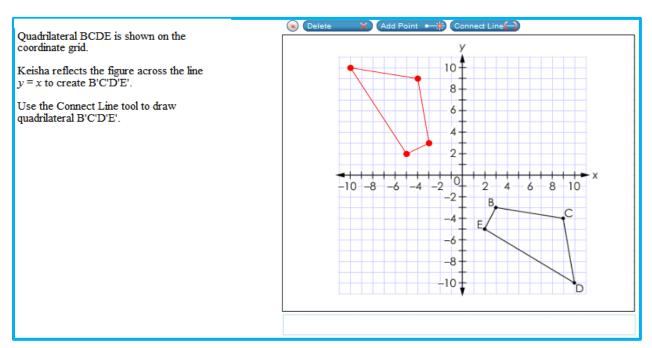
• The correct quadrilateral (1 point).

Geometry
Practice Test

**Question 2** 

**Sample Responses** 

#### Sample Response: 1 point

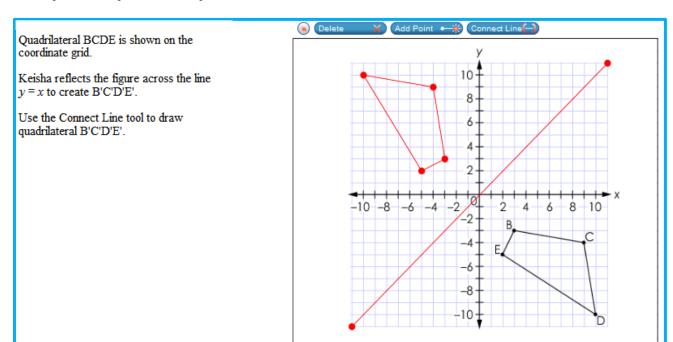


#### **Notes on Scoring**

This response earns full credit (1 point) because it shows a correct quadrilateral B'C'D'E' with the vertices B'(-3, 3), C'(-4, 9), D'(-10, 10) and E'(-5, 2).

A reflection over a line y = x is a transformation in which each point of the original quadrilateral has an image that is the same distance from the line of reflection as the original point on the opposite side of the line. The reflection of the point (x, y) across the line y = x is the point (y, x).

#### Sample Response: 1 point

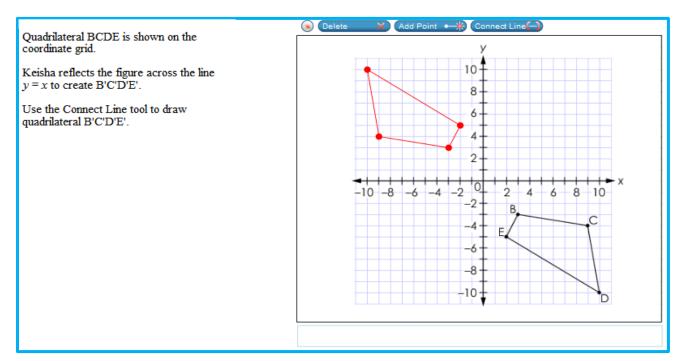


#### **Notes on Scoring**

This response earns full credit (1 point) because it shows a correct quadrilateral B'C'D'E' with the vertices B'(-3, 3), C'(-4, 9), D'(-10, 10) and E'(-5, 2) and a correct line segment belonging to the line of reflection y = x.

A reflection over a line y = x is a transformation in which each point of the original quadrilateral has an image that is the same distance from the line of reflection as the original point on the opposite side of the line. The reflection of the point (x, y) across the line y = x is the point (y, x).

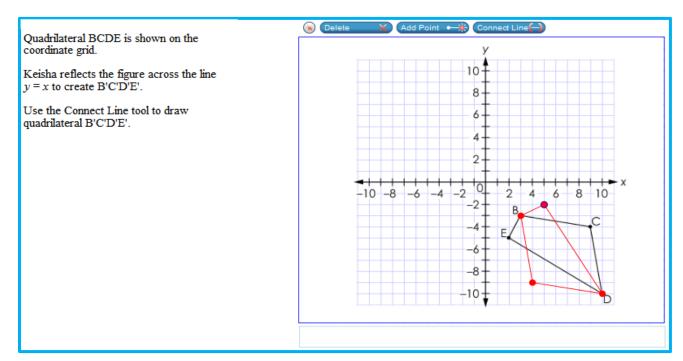
### Sample Response: 0 points



#### **Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect quadrilateral due to an extra reflection. The quadrilateral BCDE is first reflected across the line y = x and then across the line y = -x.

### Sample Response: 0 points



#### **Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect quadrilateral due to the wrong line of reflection being used. The quadrilateral BCDE is reflected across the line y = -x instead of y = x.

Geometry
Practice Test

**Question 3** 

**Question and Scoring Guidelines** 

#### **Question 3**

Three vertices of parallelogram PQRS are shown:

Q (8, 5), R (5, 1), S (2, 5)

Definition of perpendicular lines

	Statements		Reasons	
		Pythagor	an Theorem	
SR = QR		Substitut	on	
SR ≅ QR		Definitio	Definition of congruent line segments	
PS ≅ QR		Property	Property of a parallelogram	
Parallelogram PQRS is a rhombus.		Definiti	on of a rhombus	
SR = 5	$SR = \sqrt{7}$		∠PSR = 90°	
PQ = 5	$PQ = \sqrt{7}$		SR ≅ PQ	
QR = 5	$QR = \sqrt{7}$		Pythagorean Theorem	

Definition of parallel lines

Points Possible: 1

**Content Domain:** Expressing Geometric Properties with Equations

Property of a parallelogram

**Content Standard:** Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1,  $\sqrt{3}$ ) lies on the circle centered at the origin and containing the point (0, 2). (G.GPE.4)

### **Scoring Guidelines**

#### Exemplar Response

Statements	Reasons
SR = 5	Pythagorean Theorem
QR = 5	Pythagorean Theorem
SR = QR	Substitution
SR ≈ QR	Definition of congruent line segments
PS ≈ QR	Property of a parallelogram
SR ≈ PQ	Property of a parallelogram
Parallelogram PQRS is a rhombus.	Definition of a rhombus

#### Other Correct Responses

• The first two cells in the Statements column can be switched.

For this item, a full-credit response includes:

• A correctly completed table (1 point).

Geometry
Practice Test

**Question 3** 

**Sample Responses** 

#### Sample Response: 1 point

Three vertices of parallelogram PQRS are shown:				
Q (8, 5), R (5, 1), \$ (2, 5)				
Place statements and reasons in the table to complete the proof that sho	ws that parallelogram PQRS is a rhor	nbus.		
Statements			Reasons	
SR = 5		Pythagorean Theorem		
QR = 5	QR = 5 Pythagorean Theorem			
SR = QR		Substitution		
$\overline{SR} = \overline{QR}$		Definition of congruent line segments		
$\overline{PS} = \overline{QR}$		Property of a parallelogram		
SR = PQ Property of a parallelogram				
Paratlelogram PQRS is a rhombus.		Definition of a rhombus		
	$SR = \sqrt{7}$ $\angle PSR = 90^{\circ}$			
PQ = 5	$PQ = \sqrt{7}$			
	QR = √7			
Definition of perpendicular lines			Definition of parallel lines	

#### **Notes on Scoring**

This response earns full credit (1 point) because it shows a complete proof of a geometric theorem using coordinates.

A parallelogram is a rhombus if all sides are congruent. In this situation, the Pythagorean Theorem is used to calculate the side lengths of SR and QR to show that SR and QR are both 5 units. Since both pairs of opposite sides of a parallelogram are congruent (property of a parallelogram) and a pair of adjacent sides is congruent, then all four sides of the parallelogram PQRS are congruent. Thus, PQRS is a rhombus.

#### Sample Response: 1 point

Three vertices of parallelogram PQRS are shown:				
Q(8,5), R(5,1), 8(2,5)				
Place statements and reasons in the table to complete the proof that sho	ws that parallelogram PQRS is a rhor	nbus.		
Statements	Statements Reasons			
QR = 5		Pythagorean Theorem		
\$R = 5		Pythagorean Theorem		
SR = QR		Substitution		
SR ≅ QR		Definition of congruent line segments		
PS ≃ QR		Property of a parallelogram		
SR ≅ PQ		Property of a parallelogram		
Parallelogram PQRS is a rhombus.		Definition of a rhombus		
	$SR = \sqrt{7}$ $\angle PSR = 90^{\circ}$			
PQ = 5	$PQ = \sqrt{7}$			
	QR = √7			
Definition of perpendicular lines			Definition of parallel lines	

#### **Notes on Scoring**

This response earns full credit (1 point) because it shows a complete proof of a geometric theorem using coordinates.

A parallelogram is a rhombus if all sides are congruent. In this situation, the Pythagorean Theorem is used to calculate the side lengths of SR and QR to show that SR and QR are both 5 units. Since both pairs of opposite sides of a parallelogram are congruent (property of a parallelogram) and a pair of adjacent sides is congruent, then all four sides of the parallelogram PQRS are congruent. Thus, PQRS is a rhombus.

### Sample Response: 0 points

Three vertices of parallelogram PQRS are shown:

Q (8, 5), R (5, 1), S (2, 5)

Place statements and reasons in the table to complete the proof that shows that parallelogram PQRS is a rhombus.

Statements	Reasons	
QR = 5	Pythagorean Theorem	
SR = 5		
SR = QR	Substitution	
SR ≃ QR	Definition of congruent line segments	
PS ≃ QR	Property of a parallelogram	
SR ≃ PQ		
Parallelogram PQRS is a rhombus.	Definition of a rhombus	

	SR = √7	∠ PSR = 90°
PQ = 5	$PQ = \sqrt{7}$	Property of a parallelogram
	QR = √7	
Definition of perpendicular lines	Pythagorean Theorem	Definition of parallel lines
		•

#### **Notes on Scoring**

This response earns no credit (0 points) because it shows an incomplete proof of a geometric theorem using coordinates. The response is missing two reasons.

### Sample Response: 0 points

Three vertices of parallelogram PQRS are shown:				
Q (8, 5), R (5, 1), S (2, 5)				
Place statements and reasons in the table to complete the proof that shows	s that parallelogram PQRS is a rhomb	us.		
Statements			Reasons	
QR = 5		Pythagorean Theorem		
SR ≃ PQ		Property of a parallelogram		
SR = QR		Substitution		
SR ≃ QR		Definition of congruent line segments		
FS ≃ QR Property of a parallelogram				
SR = 5	SR = 5 Pythagorean Theorem			
Parallelogram PQRS is a rhombus.		Definition of a rhombus		
	$SR = \sqrt{7}$ $\angle PSR = 90^{\circ}$		∠ PSR = 90°	
PQ = 5	PQ = √7			
QR = √7				
Definition of perpendicular lines Definition of parallel lines				

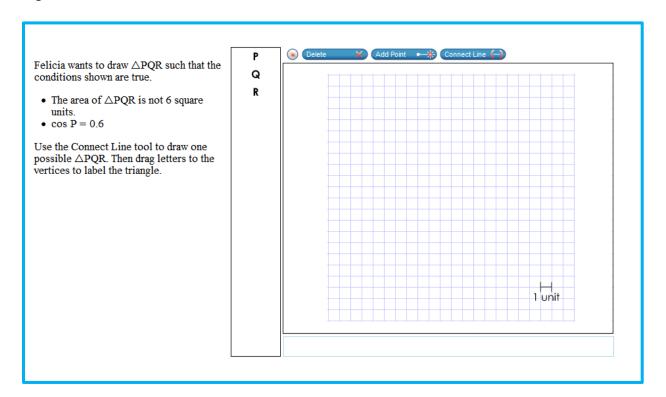
#### **Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect proof (incorrect order of statements along with incorrect justifications) of a geometric theorem using coordinates.

**Question 4** 

**Question and Scoring Guidelines** 

## **Question 4**



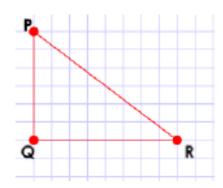
**Points Possible: 1** 

Content Domain: Similarity, Right Triangles, and Trigonometry

**Content Standard:** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. (G.SRT.6)

# **Scoring Guidelines**

#### Exemplar Response



## Other Correct Responses

• Any right triangle for which the relationship

 $\frac{the\ length\ of\ the\ leg\ adjacent\ to\ angle\ P}{hypotenuse}=0.6$ 

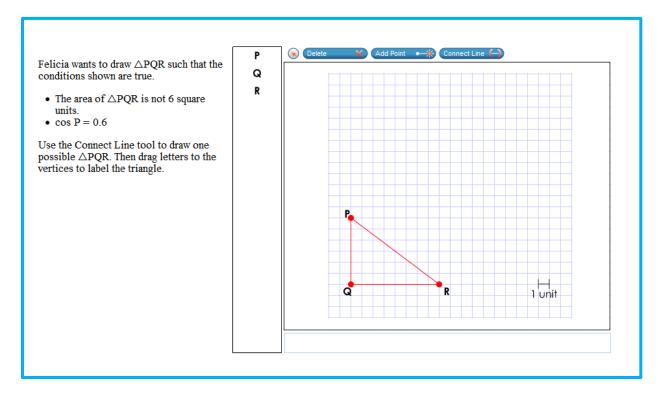
holds and whose area is not 6 square units.

For this item, a full-credit response includes:

• A correct triangle (1 point).

**Question 4** 

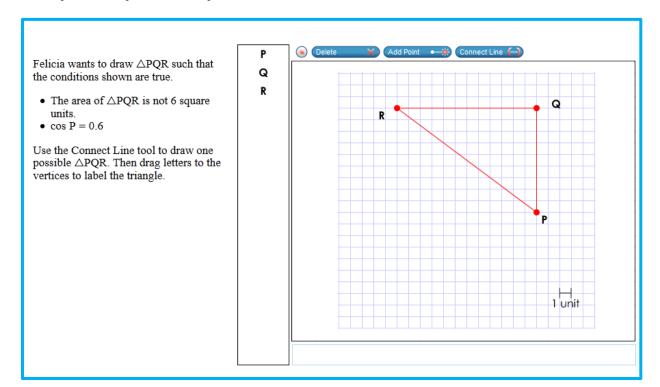
Sample Responses



#### **Notes on Scoring**

This response earns full credit (1 point) because it shows a correct triangle with  $\cos P = \frac{6}{10} \text{ or } \frac{3}{5}$  and the area  $A = \frac{1}{2} \cdot 8 \cdot 6 = 24 \text{ sq units}.$ 

This item asks students to draw and label a right triangle PQR that has an area that is not 6 sq units and where  $\cos P = 0.6$ . In right triangles, the cosine of an angle equals to the length of the adjacent leg over the length of the hypotenuse. Since  $\cos P = 0.6$ , the length of the adjacent leg/length of the hypotenuse is 0.6 or  $\frac{6}{10}$ . From here, the length of an adjacent leg can be 6 units, and the length of the hypotenuse is 10 units. Therefore, by the Pythagorean Theorem, the length of the leg that is opposite the angle P is 8 units. Drawing any right triangle with the relationship "the length of the leg adjacent to vertex P over the length of the hypotenuse equals 0.6", and with the leg adjacent to P not 3 units long, yields a correct response. A right triangle PQR with side lengths 3, 4 and 5 units long has a  $\cos P = 0.6$  and an area of 6 sq units ( $A = \frac{1}{2}bh$ ), which contradicts the given condition and, therefore, is not a correct response.

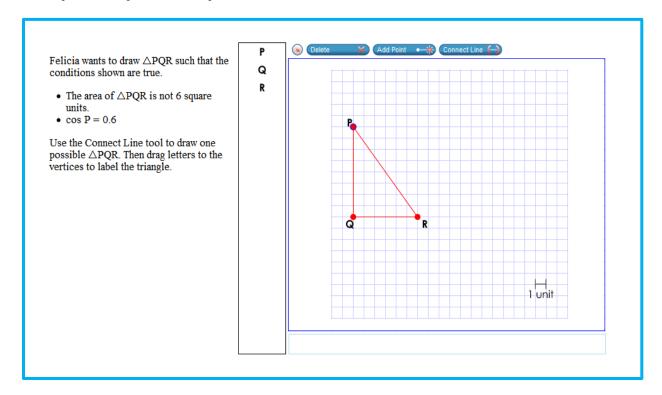


#### **Notes on Scoring**

This response earns full credit (1 point) because it shows a correct triangle with cos  $P = \frac{9}{15}$  or  $\frac{3}{5}$  and the area

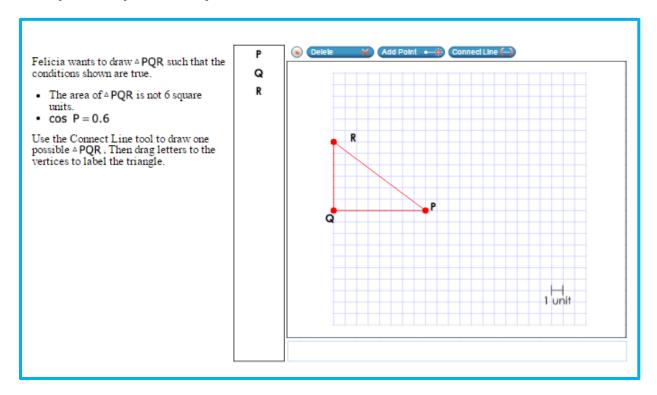
$$A = \frac{1}{2} \cdot 12 \cdot 9 = 54 \text{ sq units.}$$

This item asks students to draw and label a right triangle PQR that has an area that is not 6 sq units and where  $\cos P = 0.6$ . In right triangles, the cosine of an angle equals to the length of the adjacent leg over the length of the hypotenuse. Since  $\cos P = 0.6$ , the length of the adjacent leg/length of the hypotenuse is 0.6 or  $\frac{3}{5}$  or  $\frac{9}{15}$ . From here, the length of an adjacent leg can be 9 units, and the length of the hypotenuse is 15 units. Therefore, by the Pythagorean Theorem, the length of the leg that is opposite to the angle P is 12 units. Drawing any right triangle with the relationship "the length of the leg adjacent to vertex P over the length of the hypotenuse equals 0.6", and with the leg adjacent to P not 3 units long, yields a correct response. A right triangle PQR with side lengths 3, 4 and 5 units has a  $\cos P = 0.6$  and an area of 6 sq units  $(A = \frac{1}{2}bh)$ , which contradicts the given condition and, therefore, is not a correct response.



#### **Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect triangle with cos P =  $\frac{8}{10}$  or  $\frac{4}{5}$ , instead of  $\frac{6}{10}$ .



## **Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect triangle with  $\sin P = \frac{6}{10}$ , but  $\cos P = \frac{8}{10}$  or  $\frac{4}{5}$ , instead of  $\cos P = \frac{6}{10}$ .

**Question 5** 

**Question and Scoring Guidelines** 

## **Question 5**

Francisco asks the students in his school what pets they have. He studies the events shown.

- Event S: The student has a cat.
- Event T: The student has a dog.

Francisco finds that the two events are independent.

Select all the equations that must be true for events S and T.

- P(S|T) = P(S)
- P(S|T) = P(T)
- P(T|S) = P(S)
- P(T|S) = P(T)
- $P(S \cup T) = P(S) \bullet P(T)$
- $P(S \cap T) = P(S) \bullet P(T)$

Points Possible: 1

Content Domain: Conditional Probability and the Rules of Probability

**Content Standard:** Understand the conditional probability of A given B as P (A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. (S.CP.3)

## **Scoring Guidelines**

<u>Rationale for First Option:</u> **Key** – The student correctly identified that if the two events are independent, then the conditional probability of S given T, or  $P(S \mid T)$ , equals to a probability of S, or P(S).

<u>Rationale for Second Option:</u> This is incorrect. The student may have thought that the probability of S given T, or  $P(T \mid S)$ , is defined by the conditional event, P(T).

<u>Rationale for Third Option:</u> This is incorrect. The student may have thought that the probability of T given S, or  $P(T \mid S)$ , is defined by the probability of the conditional event S or P(S).

<u>Rationale for Fourth Option:</u> **Key** – The student correctly identified that if the two events are independent, then the conditional probability of T given S, or  $P(T \mid S)$ , must be equal to a probability of T, or P(T).

<u>Rationale for Fifth Option:</u> This is incorrect. The student may have mistaken "union" for "intersection" of the probabilities, and concluded that the probability of a union of two events  $P(T \cup S)$  is the product of probabilities,  $P(S) \cdot P(T)$ .

<u>Rationale for Sixth Option:</u> **Key** – The student correctly identified that if two events S and T are independent, then the probability of the intersection of two events, or events occurring together,  $P(S \cap T)$ , is equal to the product of their probabilities,  $P(S) \bullet P(T)$ .

**Question 5** 

Sample Responses

Francisco asks the students in his school what pets they have. He studies the events shown.

- Event S: The student has a cat.
- Event T: The student has a dog.

Francisco finds that the two events are independent.

Select all the equations that must be true for events S and T.

- P(S|T) = P(S)
- P(S|T) = P(T)
- P(T|S) = P(S)
- P(T|S) = P(T)
- $P(S \cup T) = P(S) \bullet P(T)$
- $P(S \cap T) = P(S) \bullet P(T)$

## **Notes on Scoring**

This response receives full credit (1 point) because it selects all three correct answer choices, A, D and F, and no incorrect answer choices.

Francisco asks the students in his school what pets they have. He studies the events shown.

- Event S: The student has a cat.
- Event T: The student has a dog.

Francisco finds that the two events are independent.

Select all the equations that must be true for events S and T.

- P(S|T) = P(S)
- P(S|T) = P(T)
- P(T|S) = P(S)
- P(T|S) = P(T)
- $P(S \cap T) = P(S) \cdot P(T)$

## **Notes on Scoring**

This response receives no credit (0 points) because it selects three correct and one incorrect answer choices.

Francisco asks the students in his school what pets they have. He studies the events shown.

- · Event S: The student has a cat.
- Event T: The student has a dog.

Francisco finds that the two events are independent.

Select all the equations that must be true for events S and T.

- P(S|T) = P(S)
- P(S|T) = P(T)
- P(T|S) = P(S)
- P(T|S) = P(T)
- $P(S \cap T) = P(S) \cdot P(T)$

## **Notes on Scoring**

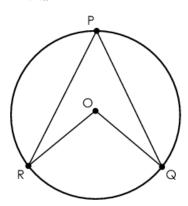
This response receives no credit (0 points) because it selects only two correct answer choices.

**Question 6** 

**Question and Scoring Guidelines** 

## Question 6

A teacher draws circle O, ∠RPQ and ∠ROQ, as shown.

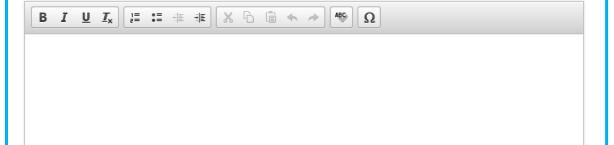


The teacher asks students to select the correct claim about the relationship between mzRPQ and mzRQQ.

- Claim 1: The measure of ∠RPQ is equal to the measure of ∠ROQ.
- Claim 2: The measure of ∠ROQ is twice the measure of ∠RPQ.

Which claim is correct? Justify your answer.

Type your answer in the space provided.



Points Possible: 1

**Content Domain:** Circles

**Content Standard:** Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. (G.C.2)

# **Scoring Guidelines**

#### Correct Responses

Claim 2 is correct because

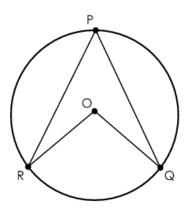
- Both angles intercept the same arc; and
- APPQ is an inscribed angle that is half the measure of the arc; and
- ROQ is a central angle that is equal to the measure of the arc.

Score Point	<u>Description</u>
1 point	Response includes the following:
	a) Claim 2 is correct
	b) Both angles ∠RPQ and ∠ROQ intercept the same arc
	c) Identification of <b>ZRPQ</b> as an inscribed angle
	d) Identification of AROQ as a central angle
0 points	The response does not meet the criteria required to earn one point. The response indicates inadequate or no understanding of the task and/or the idea or concept needed to answer the item. It may only repeat information given in the test item. The response may provide an incorrect solution/response and the provided supportive information may be irrelevant to the item, or, possibly, no other information is shown. The student may have written on a different topic or written, "I don't know".

**Question 6** 

Sample Responses

A teacher draws circle O, ∠RPQ and ∠ROQ, as shown.



The teacher asks students to select the correct claim about the relationship between mzRPQ and mzROQ.

- Claim 1: The measure of ∠RPQ is equal to the measure of ∠ROQ.
- Claim 2: The measure of ∠ROQ is twice the measure of ∠RPQ.

Which claim is correct? Justify your answer.

Type your answer in the space provided.



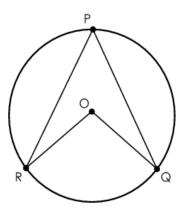
Claim 2 is correct because angle ROQ is a central angle so it is the same degree measure as the arc it intercepts. Angle RPQ is an inscribed angle so it is half the degree measure that it intercepts. Angle ROQ and angle RPQ both intercept the same arc.

#### **Notes on Scoring**

This response earns full credit (1 point) because it correctly indicates Claim 2 and shows an adequate justification for selecting this claim.

Since central angle ROQ intercepts a circular arc RQ, they have the same angular measure. Since inscribed angle RPQ intercepts the same arc RQ, its angle measure is half of the measure of the intercepted arc. Thus, the measure of angle ROQ is twice the measure of angle RPQ.

A teacher draws circle O, ∠RPQ and ∠ROQ, as shown.



The teacher asks students to select the correct claim about the relationship between mzRPQ and mzRQQ.

- Claim 1: The measure of ∠RPQ is equal to the measure of ∠ROQ.
- Claim 2: The measure of ∠ROQ is twice the measure of ∠RPQ.

Which claim is correct? Justify your answer.

Type your answer in the space provided.



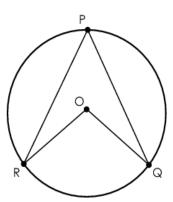
Claim 2: The measure of angle ROQ is twice the measure of angle RPQ is correct becasue an inscribed angle that has the same endpoints has another angle in the circle is half the size of the other angle. Which means the other angle is twice the size of the inscribed angle.

#### **Notes on Scoring**

This response earns full credit (1 point) because it correctly indicates Claim 2 and shows imprecise but adequate justification for selecting this claim.

Since central angle ROQ intercepts a circular arc RQ, they have the same angular measure. Since inscribed angle RPQ intercepts the same arc RQ, its angle measure is half of the measure of the intercepted arc ("an inscribed angle has the same endpoints as another angle"). Thus, the measure of angle ROQ is twice the measure of angle RPQ.

A teacher draws circle O, ∠RPQ and ∠ROQ, as shown.



The teacher asks students to select the correct claim about the relationship between mzRPQ and mzRQQ.

- Claim 1: The measure of ∠RPQ is equal to the measure of ∠ROQ.
- Claim 2: The measure of ∠ROQ is twice the measure of ∠RPQ.

Which claim is correct? Justify your answer.

Type your answer in the space provided.



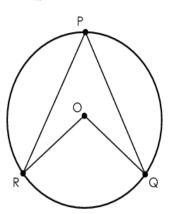
ROQ is twice RPQ because of the same points they share on the edge of the circle which makes them share the same arch but in order to have the same measure they would both have to have their point in the center so therefore ROQ is twice RPQ

#### **Notes on Scoring**

This response earns full credit (1 point) because it correctly indicates Claim 2 and shows imprecise but adequate justification for selecting this claim.

Since central angle ROQ intercepts a circular arc RQ, they have the same angular measure. Since inscribed angle RPQ intercepts the same arc RQ, its angle measure is a half of the measure of the intercepted arc ("because of the same points they share on the circle which makes them share the same arc"). Thus, the measure of angle ROQ is twice the measure of angle RPQ.

A teacher draws circle O, ∠RPQ and ∠ROQ, as shown.



The teacher asks students to select the correct claim about the relationship between mzRPQ and mzROQ.

- Claim 1: The measure of ∠RPQ is equal to the measure of ∠ROQ.
- Claim 2: The measure of ∠ROQ is twice the measure of ∠RPQ.

Which claim is correct? Justify your answer.

Type your answer in the space provided.

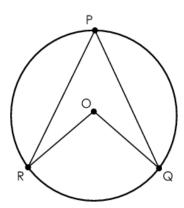


claim 2 is correct because when you make angles in a cirle than the angles in there becomes doubles because the circle gets smaller towards the top so the one angle is closer to the top so the measurements become smaller and since the oher angle is closer to the middle it gets bigger so the angle closer to the middle is bigger and is twice the size of the other angle.

#### **Notes on Scoring**

This response earns no credit (0 points) because it correctly indicates Claim 2 but shows inadequate justification for selecting this claim.

A teacher draws circle O, ∠RPQ and ∠ROQ, as shown.



The teacher asks students to select the correct claim about the relationship between mzRPQ and mzRQQ.

- Claim 1: The measure of ∠RPQ is equal to the measure of ∠ROQ.
- Claim 2: The measure of ∠ROQ is twice the measure of ∠RPQ.

Which claim is correct? Justify your answer.

Type your answer in the space provided.

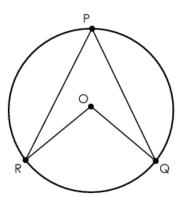


Claim one because even though RPQ is an inscribed angle only the arc is half of the inscribed angle and ROQ is just a central angle therefore, ROQ and RPQ are equal.

#### **Notes on Scoring**

This response earns no credit (0 points) because it incorrectly indicates Claim 1 and shows an incorrect justification.

A teacher draws circle O, ∠RPQ and ∠ROQ, as shown.

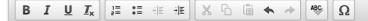


The teacher asks students to select the correct claim about the relationship between m∠RPQ and m∠ROQ.

- Claim 1: The measure of ∠RPQ is equal to the measure of ∠ROQ.
- Claim 2: The measure of ∠ROQ is twice the measure of ∠RPQ.

Which claim is correct? Justify your answer.

Type your answer in the space provided.



Claim 2 is correct because <ROQ is a central angle and <RPQ is an inscribed angle and they both are equal to the measure of arc RQ making them equal to each other.

#### **Notes on Scoring**

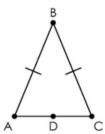
This response earns no credit (0 points) because it correctly indicates Claim 2 but shows inadequate justification for selecting this claim.

**Question 7** 

**Question and Scoring Guidelines** 

## **Question 7**

Triangle ABC is shown.



Given: Triangle ABC is isosceles. Point D is the midpoint of  $\overline{AC}$ .

Prove: ∠BAC ≅ ∠BCA

Place reasons in the table to complete the proof.

Statements	Reasons
Triangle ABC is isosceles.     D is the midpoint of AC.	1. Given
2. 2. <del>AD</del> ≅ <del>DC</del>	2. Definition of midpoint
3. 3. <del>BA</del> ≅ <del>BC</del>	3. Definition of isosceles triangle
4. 4. BD exists.	A single line segment can be drawn between any two points.
5. 5. <del>BD</del> ≅ <del>BD</del>	5.
6. 6. △ABD ≅ △CBD	6.
7. 7. ∠BAC ≅ ∠BCA	7.

AA congruency postulate	Reflexive property	
SAS congruency postulate	Symmetric property	
SSS congruency postulate Midpoint theorem		
Corresponding parts of congruent triangles are congruent.		

**Points Possible:** 1

Content Domain: Congruence

**Content Standard:** Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. (G.CO.10)

# **Scoring Guidelines**

## Exemplar Response

Statements	Reasons
Triangle ABC is isosceles.     D is the midpoint of AC,	1. Given
2. AD ≇ DC	2. Definition of midpoint
<ol> <li>BA ≅ BC</li> </ol>	3. Definition of isosceles triangle
4. BD exists.	<ol> <li>A line segment can be drawn between any two points.</li> </ol>
<ol> <li>BD ≅ BD</li> </ol>	5. Reflexive property
6. △ABD#△CBD	6. SSS postulate
<ol> <li>∠BAC≅∠BCA</li> </ol>	Corresponding parts of congruent triangles are congruent.

## Other Correct Responses

• N/A

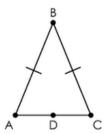
For this item, a full-credit response includes:

• A correctly completed proof (1 point).

**Question 7** 

Sample Responses

Triangle ABC is shown.



Given: Triangle ABC is isosceles. Point D is the midpoint of  $\overline{AC}$ .

Prove: ∠BAC ≅ ∠BCA

Place reasons in the table to complete the proof.

Statements	Reasons		
Triangle ABC is isosceles.     D is the midpoint of AC.	1. Given		
2. 2. <del>AD</del> ≅ <del>DC</del>	2. Definition of midpoint		
3. 3. <del>BA</del> ≅ <del>BC</del>	3. Definition of isosceles triangle		
4. 4. BD exists.	A single line segment can be drawn between any two points.		
5. 5. <del>BD</del> ≅ <del>BD</del>	5. Reflexive property		
6. 6. △ABD ≅ △CBD	6. SSS congruency postulate		
7. 7. ∠BAC ≅ ∠BCA	Corresponding parts of congruent triangles are congruent.		

AA congruency postulate

SAS congruency postulate

Symmetric property

Midpoint theorem

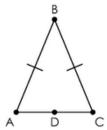
#### **Notes on Scoring**

This response earns full credit (1 point) because it shows a correct selection of reasons supporting a geometric proof about base angles of an isosceles triangle.

In this situation, the existence of three pairs of corresponding congruent sides ( $\overline{BA}$  and  $\overline{BC}$ ,  $\overline{AD}$  and  $\overline{DC}$ ,  $\overline{BD}$  and  $\overline{BD}$ ) supports the statement about triangle congruency (triangles ABD and CBD are congruent by the SSS congruency postulate). Having justified a congruency of the triangles, it follows that angles BAC and BCA are congruent because they are corresponding parts of congruent triangles.

#### Sample Response: 0 points

Triangle ABC is shown.



Given: Triangle ABC is isosceles. Point D is the midpoint of  $\overline{AC}$ .

Prove: ∠BAC ≅ ∠BCA

Place reasons in the table to complete the proof.

Statements	Reasons
Triangle ABC is isosceles.     D is the midpoint of AC.	1. Given
2. 2. <del>AD</del> ≅ <del>DC</del>	2. Definition of midpoint
3. 3. $\overline{BA} \cong \overline{BC}$	3. Definition of isosceles triangle
4. 4. BD exists.	A single line segment can be drawn between any two points.
5. 5. <del>BD</del> ≅ <del>BD</del>	5. Reflexive property
6. 6. △ABD ≅ △CBD	6. SSS congruency postulate
7. 7. ∠BAC ≅ ∠BCA	7. Symmetric property

AA congruency postulate

SAS congruency postulate

Corresponding parts of congruent triangles are congruent.

Midpoint theorem

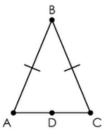
#### **Notes on Scoring**

This response earns no credit (0 points) because one of the reasons selected to support a geometric proof about base angles of an isosceles triangle is incorrect.

Angles BAC and BCA are congruent because they are corresponding parts of congruent triangles, not by the symmetric property.

## Sample Response: 0 points

Triangle ABC is shown.



Given: Triangle ABC is isosceles. Point D is the midpoint of  $\overline{AC}$ .

Prove: ∠BAC ≅ ∠BCA

Place reasons in the table to complete the proof.

Statements	Reasons
Triangle ABC is isosceles.     D is the midpoint of AC.	1. Given
2. 2. <del>AD</del> ≅ <del>DC</del>	2. Definition of midpoint
3. 3. <del>BA</del> ≅ <del>BC</del>	3. Definition of isosceles triangle
4. 4. BD exists.	A single line segment can be drawn between any two points.
5. 5. <del>BD</del> ≅ <del>BD</del>	5.
6. 6. △ABD ≅ △CBD	6. SSS congruency postulate
7. 7. ∠BAC ≅ ∠BCA	Corresponding parts of congruent triangles are congruent.

AA congruency postulate	Symmetric property
SAS congruency postulate	
Reflexive property	Midpoint theorem

#### **Notes on Scoring**

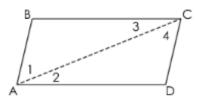
This response earns no credit (0 points) because it misses a reason  $(\overline{BD} \cong \overline{BD})$  by a reflexive property) necessary to support a geometric proof about base angles of an isosceles triangle.

**Question 8** 

**Question and Scoring Guidelines** 

#### **Question 8**

The proof shows that opposite angles of a parallelogram are congruent.



Given: ABCD is a parallelogram with diagonal AC.

Prove: ∠BAD # ∠DCB

#### Proof:

Statements	Reasons
ABCD is a parallelogram with diagonal AC.	Given
$\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$	Definition of parallelogram
∠2 ≅ ∠3 ∠1 ≅ ∠4	Alternate interior angles are congruent.
$m \angle 2 = m \angle 3$ and $m \angle 1 = m \angle 4$	Measures of congruent angles are equal.
$m \ge 1 + m \ge 2 = m \ge 4 + m \ge 2$	Addition property of equality
$m \ge 1 + m \ge 2 = m \ge 4 + m \ge 3$	?
$m \angle 1 + m \angle 2 = m \angle BAD$ $m \angle 3 + m \angle 4 = m \angle DCB$	Angle addition postulate
$m \angle BAD = m \angle DCB$	Substitution
∠BAD ≅ ∠DCB	Angles are congruent when their measures are equal.

What is the missing reason in this partial proof?

ASA

B Substitution

Angle addition postulate

Alternate interior angles are congruent.

Points Possible: 1

Content Domain: Congruence

**Content Standard:** Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. (G.CO.11)

## **Scoring Guidelines**

<u>Rationale for Option A:</u> This is incorrect. The student may have realized that ASA could be used later in the proof, but the sides have not been proved congruent yet, so this is not the correct reason for this step.

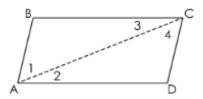
<u>Rationale for Option B:</u> **Key** – The student noticed that the previous step just had angle 3 substituted in for angle 2.

<u>Rationale for Option C:</u> This is incorrect. The student may have noted that there are angles being added, but that does not justify the current step.

<u>Rationale for Option D:</u> This is incorrect. The student may have noted that the alternate interior angles are being used, but that does not justify the current step.

## Sample Response: 1 point

The proof shows that opposite angles of a parallelogram are congruent.



Given: ABCD is a parallelogram with diagonal AC. Prove: ∠BAD ≅ ∠DCB

#### Proof:

Statements	Reasons
ABCD is a parallelogram with diagonal AC.	Given
AB    CD and AD    BC	Definition of parallelogram
∠2 ≅ ∠3 ∠1 ≅ ∠4	Alternate interior angles are congruent.
$m \angle 2 = m \angle 3$ and $m \angle 1 = m \angle 4$	Measures of congruent angles are equal.
$m \ge 1 + m \ge 2 = m \ge 4 + m \ge 2$	Addition property of equality
$m \ge 1 + m \ge 2 = m \ge 4 + m \ge 3$	?
$m \angle 1 + m \angle 2 = m \angle BAD$ $m \angle 3 + m \angle 4 = m \angle DCB$	Angle addition postulate
$m \angle BAD = m \angle DCB$	Substitution
∠BAD ≅ ∠DCB	Angles are congruent when their measures are equal.

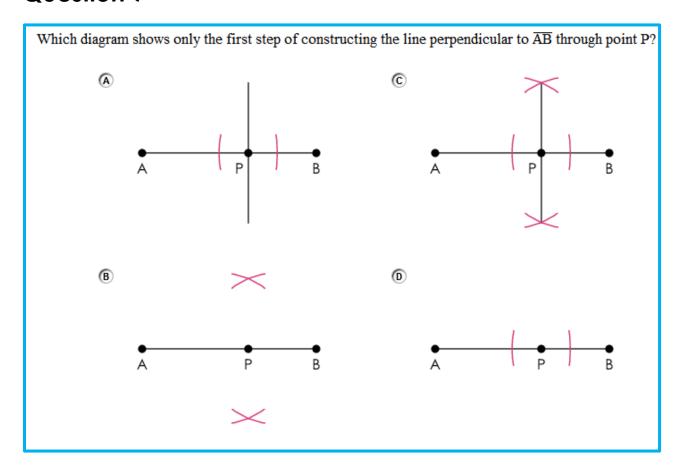
What is the missing reason in this partial proof?

- ASA
- Substitution
- Angle addition postulate
- Alternate interior angles are congruent.

**Question 9** 

**Question and Scoring Guidelines** 

#### **Question 9**



**Points Possible:** 1

Content Domain: Congruence

**Content Standard:** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. (G.CO.12)

## **Scoring Guidelines**

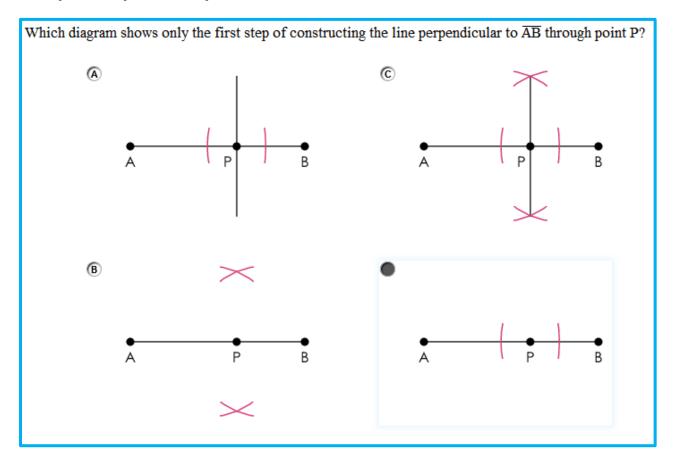
<u>Rationale for Option A:</u> This is incorrect. The student may not have realized that there are other steps between creating two points on a line segment AB that are equidistant from point P and drawing the line through point P perpendicular to the line segment AB.

<u>Rationale for Option B:</u> This is incorrect. The student may not have realized that the arc marks above and below point P cannot be constructed before constructing points on line segment AB that are equidistant from point P.

<u>Rationale for Option C:</u> This is incorrect. The student may have identified the last step instead of the first.

<u>Rationale for Option D:</u> **Key** – The student correctly identified that the first step is to create two points on line segment AB that are equidistant from point P, to use as the centers for constructing arcs above and below point P.

## Sample Response: 1 point

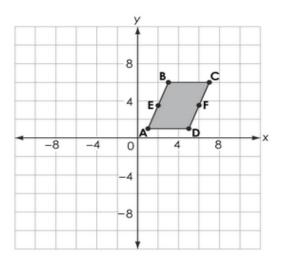


**Question 10** 

**Question and Scoring Guidelines** 

#### **Question 10**

Parallelogram ABCD is shown. Point E is the midpoint of segment AB. Point F is the midpoint of segment CD.



Which transformation carries the parallelogram onto itself?

- A a reflection across line segment AC
- a reflection across line segment EF
- a rotation of 180 degrees clockwise about the origin
- D a rotation of 180 degrees clockwise about the center of the parallelogram

**Points Possible: 1** 

Content Domain: Congruence

**Content Standard:** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. (G.CO.3)

# **Scoring Guidelines**

Rationale for Option A: This is incorrect. The student may have thought that if diagonal  $\overline{AC}$  divides ABCD into two congruent triangles, then the parallelogram would have a line of symmetry over diagonal  $\overline{AC}$ . However, since  $\overline{AC}$  is not perpendicular to  $\overline{BD}$ , vertex B will not be carried onto vertex D.

Rationale for Option B: This is incorrect. The student may have thought that since points E and F are midpoints of the sides  $\overline{AB}$  and  $\overline{CD}$ , the parallelogram has a horizontal line of symmetry. However, since  $\overline{EF}$  is not perpendicular to  $\overline{AB}$  and  $\overline{CD}$ , vertex A will not be carried onto vertex B, and vertex D will not be carried onto vertex C.

<u>Rationale for Option C:</u> This is incorrect. The student may have realized that a 180-degree rotation could carry the parallelogram onto itself, but did not take into account that this depends on where the center of rotation is. When the center of rotation is at the origin, the image of the parallelogram is in Quadrant III, meaning the image will not carry onto the pre-image.

Rationale for Option D: **Key** – The student noted that all parallelograms have 180-degree rotational symmetry about the center of the parallelogram (i.e., vertex A will be carried onto vertex C, vertex B will be carried onto vertex D, vertex C will be carried onto vertex A, and vertex D will be carried onto vertex B).

#### Sample Response: 1 point

a rotation of 180 degrees clockwise about the origin

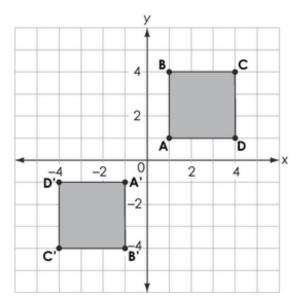
a rotation of 180 degrees clockwise about the center of the parallelogram

**Question 11** 

**Question and Scoring Guidelines** 

#### **Question 11**

Square ABCD is transformed to create the image A'B'C'D', as shown.



Select all of the transformations that could have been performed.

- $\Box$  a reflection across the line y = x
- $\Box$  a reflection across the line y = -2x
- a rotation of 180 degrees clockwise about the origin
- a reflection across the x-axis, and then a reflection across the y-axis
- a rotation of 270 degrees counterclockwise about the origin, and then a reflection across the x-axis

**Points Possible:** 1

Content Domain: Congruence

**Content Standard:** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. (G.CO.6)

## **Scoring Guidelines**

<u>Rationale for First Option:</u> This is incorrect. The student may have thought that both given figures have to be carried onto themselves by reflecting across y = x, instead of carrying ABCD onto A'B'C'D'.

Rationale for Second Option: This is incorrect. The student may have seen that the line of reflection of y = -2x would create an image of square ABCD in Quadrant III, but did not confirm that the line of reflection is a perpendicular bisector of each line segment created by connecting corresponding vertices.

Rationale for Third Option: **Key** – The student correctly identified that with a 180-degree rotation, any point (x, y) will carry onto a point (-x, -y), so that a point A (1, 1) carries onto A'(-1, -1); B (1, 4) carries onto B' (-1, -4); C(4, 4) carries onto C'(-4, -4) and D(4, 1) carries onto D'(-4, -1).

Rationale for Fourth Option: **Key** – The student correctly identified that with a reflection across the x-axis, any point (x, y) will carry onto the point (x, -y), and then, the next reflection across the y-axis, will carry any point (x, -y) onto (-x, -y). Therefore, point A (1, 1) first carries onto (1, -1) and then onto A'(-1, -1); point B (1, 4) first carries onto (1, -4) and then onto B'(-1, -4); point C(4, 4) first carries onto (4, -4) and then onto D'(-4, -1).

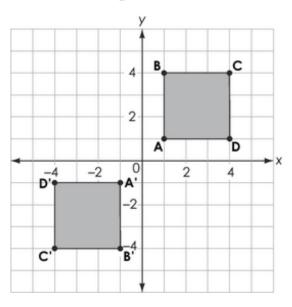
<u>Rationale for Fifth Option:</u> This is incorrect. The student may have seen that this set of transformations creates a final image in the same location as A'B'C'D' but did not see that this set of transformations does not carry the vertices in ABCD to their corresponding vertices in A'B'C'D'.

**Question 11** 

Sample Responses

#### Sample Response: 1 point

Square ABCD is transformed to create the image A'B'C'D', as shown.



Select all of the transformations that could have been performed.

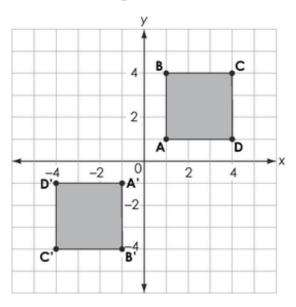
- $\Box$  a reflection across the line y = x
- a reflection across the line y = -2x
- a rotation of 180 degrees clockwise about the origin
- a reflection across the x-axis, and then a reflection across the y-axis
- a rotation of 270 degrees counterclockwise about the origin, and then a reflection across the x-axis

#### **Notes on Scoring**

This response earns full credit (1 point) because it selects both correct options, C and D, and no incorrect answer choices.

#### Sample Response: 0 points

Square ABCD is transformed to create the image A'B'C'D', as shown.



Select all of the transformations that could have been performed.

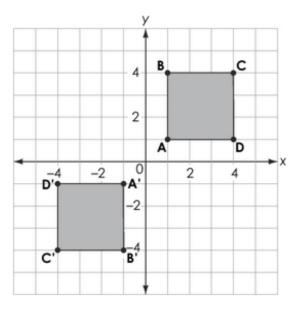
- $\square$  a reflection across the line y = x
- a reflection across the line y = -2x
- a rotation of 180 degrees clockwise about the origin
- a reflection across the x-axis, and then a reflection across the y-axis
- a rotation of 270 degrees counterclockwise about the origin, and then a reflection across the x-axis

#### **Notes on Scoring**

This response earns no credit (0 points) because it selects both correct options, C and D, and one incorrect option, A.

## Sample Response: 0 points

Square ABCD is transformed to create the image A'B'C'D', as shown.



Select all of the transformations that could have been performed.

- a reflection across the line y = x
- a reflection across the line y = -2x
- a rotation of 180 degrees clockwise about the origin
- a reflection across the x-axis, and then a reflection across the y-axis
- a rotation of 270 degrees counterclockwise about the origin, and then a reflection across the x-axis

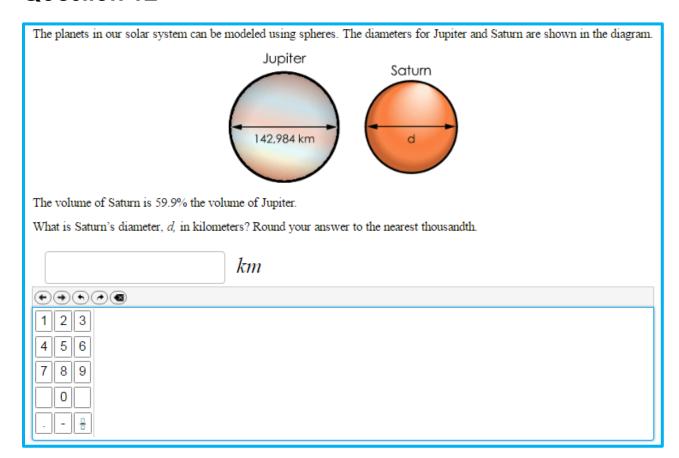
#### **Notes on Scoring**

This response earns no credit (0 points) because it selects one correct option, D, and one incorrect option, E.

**Question 12** 

**Question and Scoring Guidelines** 

## **Question 12**



Points Possible: 1

Content Domain: Geometric Measurement and Dimension

**Content Standard:** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. (G.GMD.3)

# **Scoring Guidelines**

#### Exemplar Response

• 120530.340

## Other Correct Responses

• Any value between 120530 and 120531.

For this item, a full-credit response includes:

• A correct value (1 point).

**Question 12** 

Sample Responses

#### Sample Response: 1 point

The planets in our solar system can be modeled using spheres. The diameters for Jupiter and Saturn are shown in the diagram.

Jupiter

Sciturn

Sciturn

What is Saturn's diameter, d, in kilometers? Round your answer to the nearest thousandth.

120530.340

km

\*\*Material Round\*\*

\*\*Mate

#### **Notes on Scoring**

This response earns full credit (1 point) because it shows a correct answer for Saturn's diameter, in kilometers, rounded to the nearest thousandth.

In this situation, the correct solution process uses the formula for the volume of a sphere,  $V = \frac{4}{3}(\pi \cdot r^3)$ , and the formula for the radius of a sphere being half of the diameter. A solution process may consist of two parts. In the first part, the process identifies the radius of Jupiter being half of the diameter, then uses the formula for finding the volume of Jupiter. In the second part, the process is reversed. First, it applies 59.9% to find the volume of Saturn, then it uses the formula for the volume of a sphere to find the radius of Saturn, and then it doubles the radius to find the diameter of Saturn.

#### Sample Response: 1 point

The planets in our solar system can be modeled using spheres. The diameters for Jupiter and Saturn are shown in the diagram.

Jupiter

Scaturn

Mat is Saturn's diameter, d, in kilometers? Round your answer to the nearest thousandth.

120530.000

km

Mat is Scaturn's diameter, d, in kilometers? Round your answer to the nearest thousandth.

120530.000

Name

Scaturn

Scat

#### **Notes on Scoring**

This response earns full credit (1 point) because it shows a correct answer for Saturn's diameter, in kilometers, rounded to an allowable value between 120530 and 120531.

In this situation, the correct solution process uses the formula for the volume of a sphere,  $V = \frac{4}{3}(\pi \cdot r^3)$ , and the formula for the radius of a sphere being half of the diameter. A solution process may consist of two parts. In the first part, the process identifies the radius of Jupiter being the half of the diameter, then uses the formula for finding the volume of Jupiter. In the second part, the process is reversed. First, it applies 59.9% to find the volume of Saturn, then it uses the formula for the volume of a sphere to find the radius of Saturn, and then it doubles the radius to find the diameter of Saturn.

## Sample Response: 0 points

The planets in our solar system can be modeled using spheres. The diameters for Jupiter and Saturn are shown in the diagram.

Jupiter

Soturn

Soturn

The volume of Saturn is 59.9% the volume of Jupiter.

What is Saturn's diameter, d, in kilometers? Round your answer to the nearest thousandth.

85647.416

km

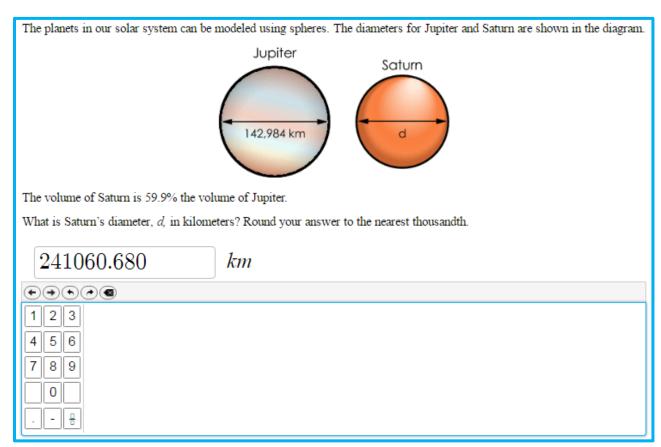
\*\*The volume of Saturn is 59.9% the volume of Jupiter.

What is Saturn's diameter, d, in kilometers? Round your answer to the nearest thousandth.

#### **Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect answer for Saturn's diameter, in kilometers, rounded to the nearest thousandth.

## Sample Response: 0 points



#### **Notes on Scoring**

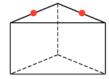
This response earns no credit (0 points) because it shows an incorrect answer for Saturn's diameter, in kilometers, rounded to the nearest thousandth.

**Question 13** 

**Question and Scoring Guidelines** 

## **Question 13**

A cross section of a right triangular prism is created by a plane cut through the points shown and is also perpendicular to the opposite base.



What is the most specific name of the shape representing the cross section?

- A triangle
- ® rectangle
- © trapezoid
- parallelogram

Points Possible: 1

Content Domain: Geometric Measurement and Dimension

**Content Standard:** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. (G.GMD.4)

# **Scoring Guidelines**

<u>Rationale for Option A:</u> This is incorrect. The student may have confused a cross section intersecting both points and being perpendicular to the opposite base with the top base of the prism being in the shape of a triangle containing both points.

<u>Rationale for Option B:</u> **Key** – The student noted that the cross section containing both points and being perpendicular to the opposite base is a quadrilateral with four right angles and congruent opposite sides, or a rectangle.

<u>Rationale for Option C:</u> This is incorrect. The student may have ignored that the cross section is perpendicular to the opposite base and incorrectly concluded that it forms a shape that has only one pair of non-congruent parallel sides and no right angles.

<u>Rationale for Option D:</u> This is incorrect. The student may have realized that the cross section has two pairs of parallel sides, but ignored that because it is perpendicular to the base, so all of the angles are right angles.

#### Sample Response: 1 point

A cross section of a right triangular prism is created by a plane cut through the points shown and is also perpendicular to the opposite base.



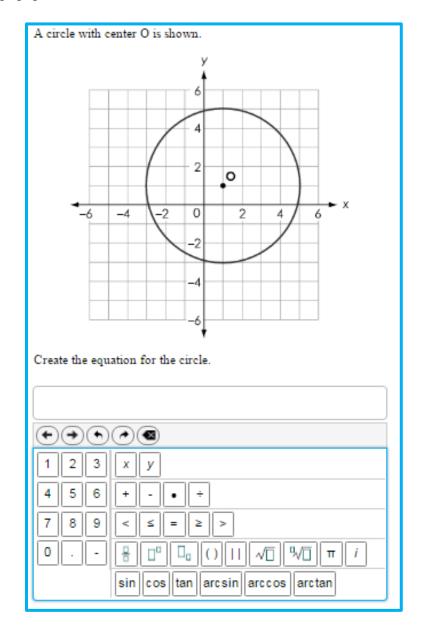
What is the most specific name of the shape representing the cross section?

- A triangle
- rectangle
- © trapezoid
- parallelogram

**Question 14** 

**Question and Scoring Guidelines** 

## **Question 14**



Points Possible: 1

**Content Domain:** Expressing Geometric Properties with Equations

**Content Standard:** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. (G.GPE.1)

# **Scoring Guidelines**

#### Exemplar Response

• 
$$(x-1)^2 + (y-1)^2 = 4^2$$

### Other Correct Responses

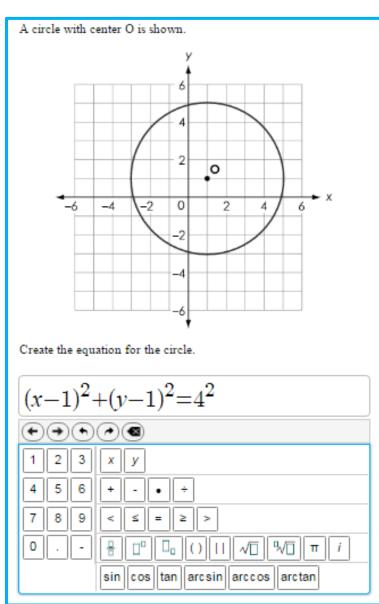
• Any equivalent equation.

For this item, a full-credit response includes:

• A correct equation (1 point).

**Question 14** 

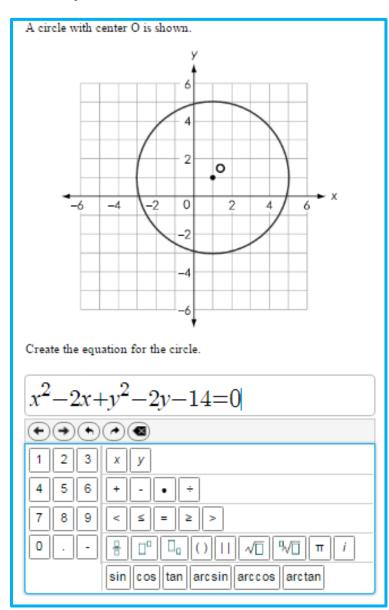
Sample Responses



#### **Notes on Scoring**

This response earns full credit (1 point) because it shows the correct center-radius form for the equation of the circle  $(x - 1)^2 + (y - 1)^2 = 4^2$ .

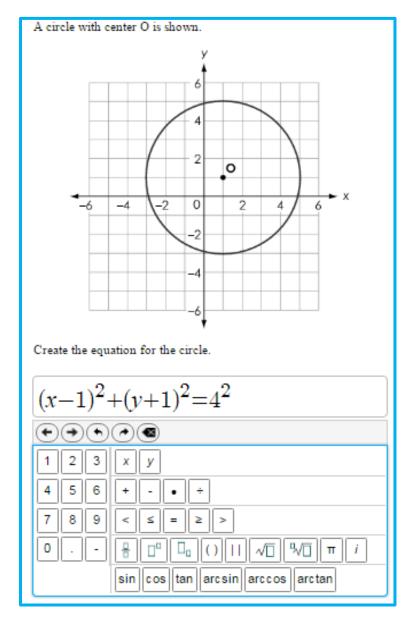
On the coordinate plane, the center-radius form for the equation of a circle with center (h, k) and radius r is  $(x - h)^2 + (y - k)^2 = r^2$ . The given circle has a center at (1, 1) and a radius of 4 units. By substituting h = 1, k = 1 and r = 4 in the center-radius form for h, k and r, respectively, the equation of the circle is  $(x - 1)^2 + (y - 1)^2 = 4^2$ , which is equivalent to the equation  $(x - 1)^2 + (y - 1)^2 = 16$ .



#### **Notes on Scoring**

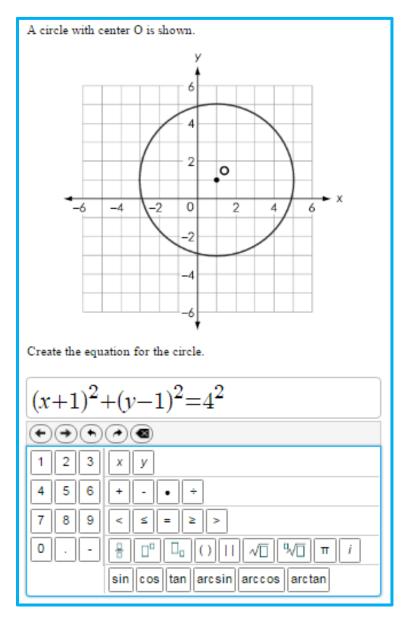
This response earns full credit (1 point) because it shows the correct general form for the equation of the circle  $(x-1)^2 + (y-1)^2 = 16$ .

On the coordinate plane, the center-radius form for the equation of a circle with center (h, k) and radius r is  $(x - h)^2 + (y - k)^2 = r^2$ . The given circle has a center at (1, 1) and a radius of 4 units. By substituting h = 1, k = 1 and r = 4 in the center-radius form for h, k and r, respectively, the equation of the circle is  $(x - 1)^2 + (y - 1)^2 = 16$ . When the equation is multiplied out and like terms are combined, the equation appears in general form,  $x^2 - 2x + y^2 - 2y - 14 = 0$ .



### **Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect center-radius form for the equation of the circle. The correct equation in center-radius form is  $(x-1)^2 + (y-1)^2 = 4^2$ .



#### **Notes on Scoring**

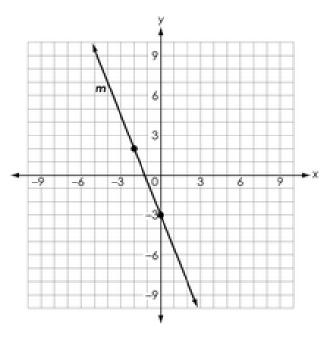
This response earns no credit (0 points) because it shows an incorrect center-radius form for the equation of the circle. The correct equation in center-radius form is  $(x-1)^2 + (y-1)^2 = 4^2$ .

**Question 15** 

**Question and Scoring Guidelines** 

## **Question 15**

The graph of line m is shown.



What is the equation of the line that is perpendicular to line m and passes through the point (3, 2)?

y =

$\oplus \oplus \bullet \bullet  $
1 2 3 x
4 5 6 + - • ÷
7 8 9 < ≤ = ≥ >
O 8 C C ()     √0 √0 π i
sin cos tan arcsin arccos arctan

Points Possible: 1

**Content Domain:** Expressing Geometric Properties with Equations

**Content Standard:** Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). (G.GPE.5)

# **Scoring Guidelines**

### Exemplar Response

### Other Correct Responses

• Any equivalent equation.

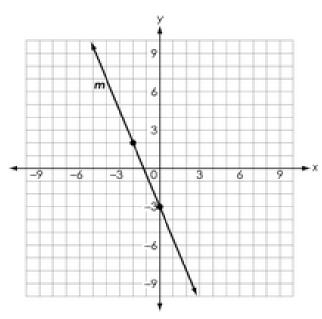
For this item, a full-credit response includes:

• A correct equation (1 point).

**Question 15** 

Sample Responses

The graph of line m is shown.



What is the equation of the line that is perpendicular to line m and passes through the point (3, 2)?

$$y = \sqrt{\frac{2}{5}x + \frac{4}{5}}$$

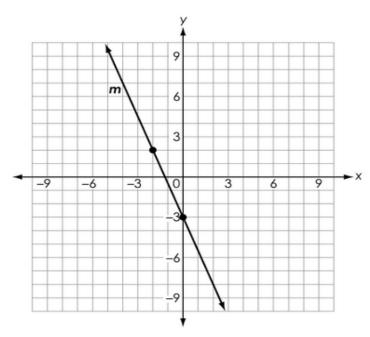
- (+)(+)(+)(**@**)
- 1 2 3 x
- 4 5 6 + · ÷
- 7 8 9 < ≤ = ≥ >
- 0. 8 0° 0 () Π √0 %0 π i
  - sin cos tan arcsin arccos arctan

#### **Notes on Scoring**

This response earns full credit (1 point) because it shows a correct equation of a line perpendicular to a given line that passes through a given point.

For this situation, the student can find the slope-intercept form of the equation of the line to get the correct answer. The slope of any line perpendicular to the given line is  $\frac{2}{5}$  because it is the opposite reciprocal of the slope of line m,  $-\frac{5}{2}$ . If the slope of a perpendicular line,  $\frac{2}{5}$ , and the point it passes through, (3, 2), are substituted back into the slope-intercept form y = mx + b, the equation becomes  $2 = \frac{2}{5} \cdot 3 + b$ . From here,  $b = \frac{4}{5}$ , and the y-intercept of the perpendicular line is located at  $(0, \frac{4}{5})$ . The equation for the perpendicular line is then  $y = \frac{2}{5} \cdot x + \frac{4}{5}$ .

The graph of line m is shown.



What is the equation of the line that is perpendicular to line m and passes through the point (3, 2)?

y = 0.4(x-3)+2

- (+)(+)(+)(<u>\*</u>)
- 1 2 3 x
- 4 5 6 + ÷
- 7 8 9 < ≤ = ≥ >
- 0 . Η Ο Ο () | | √Ο ΨΟ π i

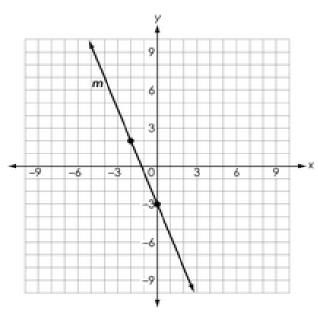
sin cos tan arcsin arccos arctan

#### **Notes on Scoring**

This response earns full credit (1 point) because it shows a correct equivalent equation of a line perpendicular to a given line that passes through a given point.

For this situation, the student can solve the point-slope form of the equation of the perpendicular line for y to get the correct answer. The slope of any line perpendicular to the given line is  $\frac{2}{5}$ , because it is the opposite reciprocal of the slope of line m,  $-\frac{5}{2}$ . If the slope of a perpendicular line,  $\frac{2}{5}$  or 0.4, and the point it passes through, (3, 2), are substituted back into the slope-point form  $y - y_1 = m(x - x_1)$ , the form becomes y - 2 = 0.4(x - 3), and then y = 0.4(x - 3) + 2.

The graph of line m is shown.



What is the equation of the line that is perpendicular to line m and passes through the point (3, 2)?

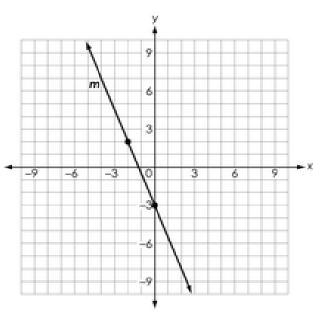
$$y = \sqrt{-\frac{5}{2}x + \frac{25}{2}}$$

$\odot \odot \odot \odot \odot$
1 2 3 x
4 5 6 + - • ÷
7 8 9 < ≤ = ≥ >
O 8 C C () [ [ √□ [ √□ [ π ] ]
sin cos tan arcsin arccos arctan

## **Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect equation of the line perpendicular to a given line that passes through a given point.

The graph of line m is shown.



What is the equation of the line that is perpendicular to line m and passes through the point (3, 2)?

$$y = \left[\frac{2}{5}x + \frac{8}{5}\right]$$

$\oplus \oplus \oplus \oplus \otimes$
1 2 3 x
4 5 8 + - • ÷
7 8 9 < < = ≥ >
0 # C° C () [] √0 5√0 π i
sin cos tan arcsin arccos arctan

### **Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect equation of the line perpendicular to a given line that passes through a given point.

**Question 16** 

**Question and Scoring Guidelines** 

# **Question 16**

Line segment AC has endpoints A $(-1, -3.5)$ and C $(5, -1)$ .
Point B is on line segment AC and is located at $(0.2, -3)$ .
What is the ratio of $\frac{AB}{BC}$ ?
$\bullet \bullet \bullet \bullet \otimes$
1 2 3
4 5 6
789

**Points Possible:** 1

**Content Domain:** Expressing Geometric Properties with Equations

**Content Standard:** Find the point on a directed line segment between two given points that partitions the segment in a given ratio. (G.GPE.6)

# **Scoring Guidelines**

#### Exemplar Response

 $\bullet$   $\frac{1}{4}$ 

## Other Correct Responses

• Any equivalent value.

For this item, a full-credit response includes:

• A correct ratio (1 point).

**Question 16** 

Sample Responses

Line segment AC has endpoints A $(-1, -3.5)$ and C $(5, -1)$ .
Point B is on line segment AC and is located at $(0.2, -3)$ .
What is the ratio of $\frac{AB}{BC}$ ?
$\left[rac{1}{4} ight]$
$\bullet \bullet \bullet \bullet \otimes$
123
4 5 6
789
🖶

### **Notes on Scoring**

This response earns full credit (1 point) because it shows a correct ratio of  $\frac{AB}{BC}$  or  $\frac{1}{4}$ .

One of several ways to approach this situation is to use a distance formula,  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ , by substituting the coordinates of two points to find the length of a line segment AB and the length a line segment BC. Since AB = 1.3 and BC = 5.2, the ratio of  $\frac{AB}{BC}$  is  $\frac{1.3}{5.2}$  or  $\frac{1}{4}$ .

Line segment AC has endpoints A $(-1, -3.5)$ and C $(5, -1)$ .
Point B is on line segment AC and is located at $(0.2, -3)$ .
What is the ratio of $\frac{AB}{BC}$ ?
$\boxed{0.25}$
$\bullet \bullet \bullet \bullet \otimes$
1 2 3
4 5 6
789
=

### **Notes on Scoring**

This response earns full credit (1 point) because it shows a correct ratio of  $\frac{AB}{BC}$  or  $\frac{1}{4}$  or .25.

One of several ways to approach this situation is to use a distance formula,  $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ , by substituting the coordinates of two points to find the length of a line segment AB and the length of a line segment BC. Since AB = 1.3 and BC = 5.2, the ratio of  $\frac{AB}{BC}$  is  $\frac{1.3}{5.2}$  or  $\frac{1}{4}$ .

#### **Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect ratio of  $\frac{AB}{BC}$  as  $\frac{1}{6}$ .

Line segment AC has endpoints A $(-1, -3.5)$ and C $(5, -1)$ .					
Point B is on line segment AC and is located at (0.2, -3).					
What is the ratio of $\frac{AB}{BC}$ ?					
$\lfloor 4 \rfloor$					
$\bullet \bullet \bullet \bullet \otimes$					
1 2 3					
4 5 6					
7 8 9					

# **Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect ratio of  $\frac{AB}{BC}$  as 4.

**Question 17** 

**Question and Scoring Guidelines** 

## **Question 17**

Triangle ABC has vertices at $(-4, 0)$ , $(-1, 6)$ and $(3, -1)$ .
What is the perimeter of triangle ABC, rounded to the nearest tenth?
$\bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet \bullet$
1 2 3
4 5 6
7 8 9
🖶

**Points Possible:** 1

**Content Domain:** Expressing Geometric Properties with Equations

**Content Standard:** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. (G.GPE.7)

# **Scoring Guidelines**

#### Exemplar Response

• 21.8

#### Other Correct Responses

• Any number greater than or equal to 21.7 and less than or equal to 22.

For this item, a full-credit response includes:

• A correct value (1 point).

**Question 17** 

Sample Responses

Triangle ABC has vertices at (-4, 0), (-1, 6) and (3, -1).

What is the perimeter of triangle ABC, rounded to the nearest tenth?

21.8

1 2 3
4 5 6
7 8 9
0

#### **Notes on Scoring**

This response earns full credit (1 point) because it shows a correct value for the perimeter of triangle ABC, rounded to the nearest tenth.

The perimeter of triangle ABC is the sum of the three side lengths. The side lengths can be found by substituting the coordinates of the end points into the distance formula,  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}, \text{ two at a time. The length of } \frac{\overline{AB}}{\overline{BC}} \text{ is } \sqrt{(-4+1)^2 + (0-6)^2} \text{ or } \sqrt{45}; \text{ the length of } \frac{\overline{BC}}{\overline{AC}} \text{ is } \sqrt{(-4-3)^2 + (6+1)^2} \text{ or } \sqrt{50}.$ 

The sum of the three side lengths is approximately 21.841529 or 21.8 rounded to the nearest tenth. Answers between 21.7 and 22 are accepted to allow for minor rounding errors.

Triangle ABC has vertices at $(-4, 0)$ , $(-1, 6)$ and $(3, -1)$ .
What is the perimeter of triangle ABC, rounded to the nearest tenth?
21.9
$\bullet \bullet \bullet \bullet \otimes$
1 2 3
4 5 6
7 8 9
8

#### **Notes on Scoring**

This response earns full credit (1 point) because it shows a correct allowed value for the perimeter of triangle ABC, rounded to the nearest tenth that is greater than or equal to 21.7 and less than or equal to 22.

The perimeter of triangle ABC is the sum of the three side lengths. The side lengths can be found by substituting the coordinates of the end points into the distance formula,  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ , two at a time. The length of  $\overline{AB}$  is  $\sqrt{(-4+1)^2 + (0-6)^2}$  or  $\sqrt{45}$ ; the length of  $\overline{BC}$  is  $\sqrt{(-1-3)^2 + (6+1)^2}$  or  $\sqrt{65}$ ; and the length of  $\overline{AC}$  is  $\sqrt{(-4-3)^2 + (0+1)^2}$  or  $\sqrt{50}$ .

The sum of the three side lengths is approximately 21.841529 or 21.8 rounded to the nearest tenth. Answers between 21.7 and 22 are accepted to allow for minor rounding errors.

Triangle ABC has vertices at (-4, 0), (-1, 6) and (3, -1).

What is the perimeter of triangle ABC, rounded to the nearest tenth?

21.84



1 2 3

4 5 6

7 8 9

0

. - 🖶

#### **Notes on Scoring**

This response earns full credit (1 point) because it shows a correct allowed value for the perimeter of triangle ABC that is greater than or equal to 21.7 and less than or equal to 22.

The perimeter of triangle ABC is the sum of the three side lengths. The side lengths can be found by substituting the coordinates of the end points into the distance formula,  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}, \text{ two at a time. The length of } AB \text{ is } \sqrt{(-4+1)^2 + (0-6)^2} \text{ or } \sqrt{45}; \text{ the length of } BC \text{ is } \sqrt{(-1-3)^2 + (6+1)^2} \text{ or } \sqrt{65}; \text{ and the length of } AC \text{ is } \sqrt{(-4-3)^2 + (0+1)^2} \text{ or } \sqrt{50}.$ 

The sum of the three side lengths is approximately 21.841529 or 21.8 rounded to the nearest tenth. Answers between 21.7 and 22 are accepted to allow for minor rounding errors and more precise answers.

Triangle ABC has vertices at $(-4, 0)$ , $(-1, 6)$ and $(3, -1)$ .
What is the perimeter of triangle ABC, rounded to the nearest tenth?
28
$\bullet \bullet \bullet \bullet \otimes$
1 2 3
4 5 6
7 8 9
8

### **Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect value for a perimeter of a triangle ABC that falls outside of the allowable range of values.

Triangle ABC has vertices at $(-4, 0)$ , $(-1, 6)$ and $(3, -1)$ .
What is the perimeter of triangle ABC, rounded to the nearest tenth?
23
1 2 3
4 5 6
7 8 9

### **Notes on Scoring**

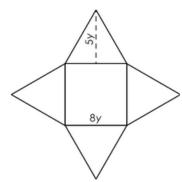
This response earns no credit (0 points) because it shows an incorrect value for a perimeter of a triangle ABC that falls outside of the allowable range of values.

**Question 18** 

**Question and Scoring Guidelines** 

### **Question 18**

Allison designs fancy boxes to fill with chocolates. The boxes are in the shape of a right square pyramid as shown, where 8y represents the length of one side of the base of the pyramid, and 5y represents the height of one triangular face of the pyramid.



The large size box must be designed to have a volume of 1,000 cubic centimeters.

- A. Create an equation that can be used to calculate the length of the base and height of the triangular face of the box. Enter your equation in the first response box.
- B. What dimensions for the length, in centimeters, of the base and the height of the triangular face, in centimeters, satisfy these constraints?

A.

- B. Length of Base = centimeters
- B. Height of Triangular Face = centimeters

$lackbox{}{\bullet}$	◆ ■
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4 5 6	+ - • ÷
7 8 9	< \leq = \geq >
0	
	sin cos tan arcsin arccos arctan

**Points Possible: 2** 

Content Domain: Modeling with Geometry

**Content Standard:** Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). (G.MG.3)

# **Scoring Guidelines**

#### **Exemplar Response**

- A.  $1000 = 64y^3$
- B. Length of Base = 20
- B. Height of Triangular Face = 12.5

#### Other Correct Responses

- Any equivalent equation for Part A.
- Any equivalent values for Part B.

For this item, a full-credit response includes:

A correct equation for Part A (1 point);

**AND** 

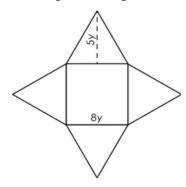
• A correct set of values for Part B (1 point).

**Note:** Students receive 1 point if their answer for Part A is equivalent to  $1000 = \frac{1}{3}(8y)^2 \cdot 5y$ , and if their answer for Part B is correct based off of this incorrect equation.

**Question 18** 

Sample Responses

Allison designs fancy boxes to fill with chocolates. The boxes are in the shape of a right square pyramid as shown, where 8y represents the length of one side of the base of the pyramid, and 5y represents the height of one triangular face of the pyramid.



The large size box must be designed to have a volume of 1,000 cubic centimeters.

- A. Create an equation that can be used to calculate the length of the base and height of the triangular face of the box. Enter your equation in the first response box.
- B. What dimensions for the length, in centimeters, of the base and the height of the triangular face, in centimeters, satisfy these constraints?

A. 
$$1000=64y^3$$

B. Length of Base = 20 centimeters

B. Height of Triangular Face = 12.5 centimeters

<b>(+)</b> (*)	→ ②
1 2 3	у
4 5 6	+ - • ÷
7 8 9	< \leq = \geq >
0	
	sin cos tan arcsin arccos arctan

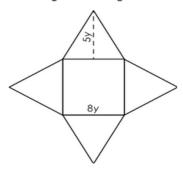
#### **Notes on Scoring**

This response earns full credit (2 points) because it shows a correct equation that can be used to calculate the volume of the square pyramid and the two correct values for the length of the base and the height of the triangular face.

The formula for the volume of a square pyramid is  $V = \frac{1}{3}Bh$ , where B is the area of the square base and h is the height of the pyramid. Since the length of the square base is 8y, the area of the square base is  $(8y)^2 = 64y^2$ . A cross-section of a pyramid that is created by a plane cut through the apex and that is perpendicular to the base forms an isosceles triangle. A half of this triangle is a right triangle with one leg being the height of a pyramid; another leg is half of a side of the square base, 4y, and the hypotenuse is the height of the triangular face, 5y. Dimensions of this right triangle are 3y, 4y and 5y (Pythagorean triple), where 3y is the height of the pyramid. Thus, an equation representing the volume of the pyramid is  $1000 = \frac{1}{3} \cdot 64y^2 \cdot 3y$  or  $64y^3 = 1000$ .

The correct solution to this equation is y = 2.5. Using this value, the length of the base is 8y or  $8 \cdot 2.5 = 20$  cm, and the height of the triangular face is 5y or 12.5 cm.

Allison designs fancy boxes to fill with chocolates. The boxes are in the shape of a right square pyramid as shown, where 8y represents the length of one side of the base of the pyramid, and 5y represents the height of one triangular face of the pyramid.



The large size box must be designed to have a volume of 1,000 cubic centimeters.

- A. Create an equation that can be used to calculate the length of the base and height of the triangular face of the box. Enter your equation in the first response box.
- B. What dimensions for the length, in centimeters, of the base and the height of the triangular face, in centimeters, satisfy these constraints?

A. 
$$\frac{1}{3}(8y)^2(3y) = 1000$$

B. Length of Base =  $8\sqrt[3]{\frac{1000}{64}}$ B. Height of Triangular Face =  $5\sqrt[3]{\frac{1000}{64}}$ centimeters

centimeters

<b>(+)</b> (+)	
1 2 3	у
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7 8 9	< \leq = \geq >
0	
,	sin cos tan arcsin arccos arctan

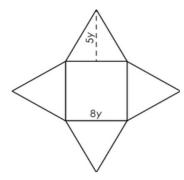
#### **Notes on Scoring**

This response earns full credit (2 points) because it shows an equivalent equation for the correct equation that can be used to calculate the volume of the square pyramid and the two equivalent values for the correct length of the base and the correct height of the triangular face.

The formula for the volume of a square pyramid is  $V = \frac{1}{3}Bh$ , where B is the area of the square base and h is the height of the pyramid. Since the length of the square base is 8y, the area of the square base is  $(8y)^2 = 64y^2$ . A cross-section of a pyramid that is created by a plane cut through the apex and that is perpendicular to the base forms an isosceles triangle. A half of this triangle is a right triangle with one leg being the height of a pyramid; another leg is half of a side of the square base, 4y, and the hypotenuse is the height of the triangular face, 5y. Dimensions of this right triangle are 3y, 4y and 5y (Pythagorean triple), where 3y is the height of the pyramid. Thus, an equation representing the volume of the pyramid is  $10000 = \frac{1}{3} \bullet (8y)^2 \bullet 3y$ .

A correct solution to this equation is  $y = \sqrt[3]{\frac{1000}{64}}$ . Using this value, the length of the base is 8y or  $8 \cdot (\sqrt[3]{\frac{1000}{64}})$ , and the height of the triangular face is 5y or  $5 \cdot (\sqrt[3]{\frac{1000}{64}})$ .

Allison designs fancy boxes to fill with chocolates. The boxes are in the shape of a right square pyramid as shown, where 8y represents the length of one side of the base of the pyramid, and 5y represents the height of one triangular face of the pyramid.



The large size box must be designed to have a volume of 1,000 cubic centimeters.

- A. Create an equation that can be used to calculate the length of the base and height of the triangular face of the box. Enter your equation in the first response box.
- B. What dimensions for the length, in centimeters, of the base and the height of the triangular face, in centimeters, satisfy these constraints?

$$A. \ \ \frac{(8y)^2(3y)}{3} = 1000$$

B. Length of Base = 2.5 centimeters

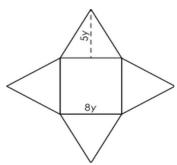
B. Height of Triangular Face = 2.5 centimeters

<b>← → ( h</b> )	$\bigcirc \bigcirc \bigcirc$
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4 5 6	+ - • ÷
7 8 9	< \leq = \geq >
0	
	sin cos tan arcsin arccos arctan

#### **Notes on Scoring**

This response earns partial credit (1 point) because it shows an equation equivalent to the correct equation that can be used to calculate the volume of the square pyramid. The length of the base and the height of the triangular face are incorrect.

Allison designs fancy boxes to fill with chocolates. The boxes are in the shape of a right square pyramid as shown, where 8y represents the length of one side of the base of the pyramid, and 5y represents the height of one triangular face of the pyramid.



The large size box must be designed to have a volume of 1,000 cubic centimeters.

- A. Create an equation that can be used to calculate the length of the base and height of the triangular face of the box. Enter your equation in the first response box.
- B. What dimensions for the length, in centimeters, of the base and the height of the triangular face, in centimeters, satisfy these constraints?

A. 
$$\left[\frac{1}{3}(8y)^2(5y)=1000\right]$$

B. Length of Base =  $8\sqrt[3]{\frac{75}{8}}$ B. Height of Triangular Face =  $5\sqrt[5]{\frac{75}{8}}$ centimeters

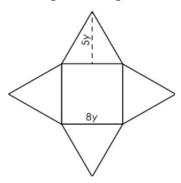
centimeters

<b>(+)</b> (+)(*)	<b>8</b>
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#### **Notes on Scoring**

This response earns partial credit (1 point) because it shows an incorrect equation (the height of the triangular face is mistakenly used instead of the height of the pyramid) to calculate the volume of a square pyramid, but correctly shows the length of the base and the height of the triangular face, based on this incorrect equation.

Allison designs fancy boxes to fill with chocolates. The boxes are in the shape of a right square pyramid as shown, where 8y represents the length of one side of the base of the pyramid, and 5y represents the height of one triangular face of the pyramid.



The large size box must be designed to have a volume of 1,000 cubic centimeters.

- A. Create an equation that can be used to calculate the length of the base and height of the triangular face of the box. Enter your equation in the first response box.
- B. What dimensions for the length, in centimeters, of the base and the height of the triangular face, in centimeters, satisfy these constraints?

$$A. \ \ \frac{1}{3}(8y)^2(5y) = 1000$$

B. Length of Base = 2.1 centimeters

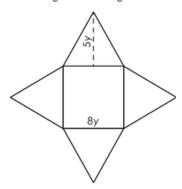
B. Height of Triangular Face = 2.1 centimeters

<b>(•)</b> (•) (•) (•)
1 2 3 y
456+-•÷
7 8 9 < ≤ = ≥ >
sin cos tan arcsin arccos arctan

#### **Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect equation and incorrect values for the lengths of the base and the height of the triangular face.

Allison designs fancy boxes to fill with chocolates. The boxes are in the shape of a right square pyramid as shown, where 8y represents the length of one side of the base of the pyramid, and 5y represents the height of one triangular face of the pyramid.



The large size box must be designed to have a volume of 1,000 cubic centimeters.

- A. Create an equation that can be used to calculate the length of the base and height of the triangular face of the box. Enter your equation in the first response box.
- B. What dimensions for the length, in centimeters, of the base and the height of the triangular face, in centimeters, satisfy these constraints?

A.  $8y \cdot 5y = 1000$ 

B. Length of Base = 2.1 centimeters

B. Height of Triangular Face = 2.1 centimeters

<b>€ (</b> •) (•)	<b>→ ③</b>
1 2 3	у
4 5 6	+ - • ÷
7 8 9	< \leq = \geq >
0	
	sin cos tan arcsin arccos arctan

#### **Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect equation and incorrect values for the lengths of the base and the height of the triangular face.

**Question 19** 

**Question and Scoring Guidelines** 

### **Question 19**

Kyle performs a transformation on a triangle. The resulting triangle is similar but not congruent to the original triangle.

Which transformation did Kyle perform on the triangle?

- A dilation
- B reflection
- © rotation
- ① translation

Points Possible: 1

**Content Domain:** Similarity, Right Triangles, and Trigonometry

**Content Standard:** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. (G.SRT.2)

# **Scoring Guidelines**

<u>Rationale for Option A:</u> **Key** – The student noted that dilation will preserve the shape and orientation (measures of all angles remain the same) but may change the side lengths proportionally, making the triangles not congruent.

<u>Rationale for Option B:</u> This is incorrect. The student may have thought that since a reflection can change orientation, that would make the two triangles not congruent, not remembering that orientation does not affect the congruence of two shapes.

<u>Rationale for Option C:</u> This is incorrect. The student may have thought that since a rotation can change the placement of an objection, that would make the two triangles not congruent, not remembering that placement does not affect the congruence of two triangles.

<u>Rationale for Option D:</u> This is incorrect. The student may have selected an option that definitely would produce congruence rather than one that would not produce congruence.

#### Sample Response: 1 point

Kyle performs a transformation on a triangle. The resulting triangle is similar but not congruent to the original triangle.

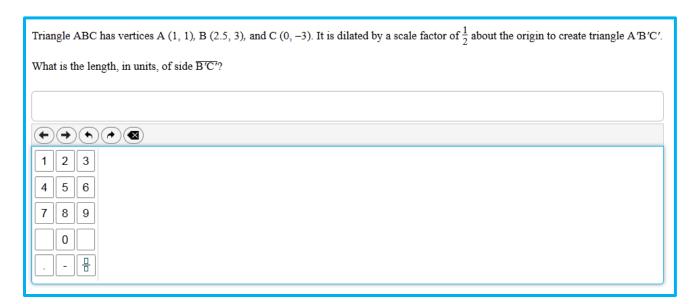
Which transformation did Kyle perform on the triangle?

- dilation
- B reflection
- © rotation
- ① translation

**Question 20** 

**Question and Scoring Guidelines** 

#### **Question 20**



Points Possible: 1

Content Domain: Similarity, Right Triangles, and Trigonometry

**Content Standard:** Verify experimentally the properties of dilations given by a center and a scale factor:

b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. (G.SRT.1b)

## **Scoring Guidelines**

#### **Exemplar Response**

• 3.25

#### Other Correct Responses

Any equivalent value.

For this item, a full-credit response includes:

• The correct length (1 point).

**Question 20** 

Sample Responses

Triangle ABC has vertices A (1, 1), B (2.5, 3), and C (0, -3). It is dilated by a scale factor of $\frac{1}{2}$ about the origin to create triangle A'B'C'.			
What is the length, in units, of side $\overline{B'C'}$ ?			
$\boxed{3.25}$			
1 2 3			
4 5 6			
7 8 9			

#### **Notes on Scoring**

This response earns full credit (1 point) because it shows the correct length of side  $\overline{B'C'}$ .

When a triangle is dilated by a positive scale factor of  $\frac{1}{2}$ , all side lengths change by this scale factor, regardless of the center of dilation. The distance formula and coordinates of points B (2.5, 3) and C (0, -3) can be used to calculate the length of a side  $\overline{BC}$  as  $\sqrt{(2.5-0)^2+(3+3)^2}=6.5$ . By applying the scale factor  $\frac{1}{2}$  to the length of BC, the length

of side B'C' = 
$$\frac{1}{2}$$
 • BC = 3.25 units.

Triangle ABC has vertices A (1, 1), B (2.5, 3), and C (0, -3). It is dilated by a scale factor of $\frac{1}{2}$ about the origin to create triangle A'B'C'.			
What is the length, in units, of side $\overline{B'C'}$ ?			
$\frac{13}{4}$			

#### **Notes on Scoring**

This response earns full credit (1 point) because it shows the correct length of side  $\overline{B'C'}$ .

When a triangle is dilated by a positive scale factor of  $\frac{1}{2}$ , all side lengths change by this scale factor, regardless of the center of dilation. The distance formula and coordinates of points B (2.5, 3) and C (0, -3) can be used to calculate the length of side  $\overline{BC}$  as  $\sqrt{(2.5-0)^2 + (3+3)^2} = 6.5$ . By applying the scale factor  $\frac{1}{2}$  to the length of BC, the length of side B'C' =  $\frac{1}{2} \cdot BC = 3.25 = \frac{13}{4}$  units.

Triangle ABC has vertices A (1, 1), B (2.5, 3), and C (0, -3). It is dilated by a scale factor of $\frac{1}{2}$ about the origin to create triangle A'B'C'.			
What is the length, in units, of side B'C'?			
6.	5		
•	•	•	)• <b>③</b>
1	2	3	
4	5	6	
7	8	9	
	0		
Ŀ	-		

## **Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect length of side  $\overline{B'C'}$ .

# Notes on Scoring

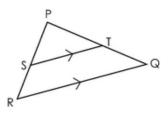
This response earns no credit (0 points) because it shows an incorrect length of side  $\overline{B'C'}$ .

**Question 21** 

**Question and Scoring Guidelines** 

## **Question 21**

Triangle PQR is shown, where  $\overline{ST}$  is parallel to  $\overline{RQ}$ .



Marta wants to prove that  $\frac{SR}{PS} = \frac{TQ}{PT}$ .

Place a statement or reason in each blank box to complete Marta's proof.

Statements	Reasons
1. <u>ST</u> ∥ <u>RQ</u>	1. Given
2. ∠PST ≅ ∠R and ∠PTS ≅ ∠Q	If two parallel lines are cut by a transversal, then corresponding angles are congruent.
3. △PQR ~ △PTS	3.
4.	4.
5. $PR = PS + SR$ , $PQ = PT + TQ$	5. Segment addition postulate
$6. \frac{PS + SR}{PS} = \frac{PT + TQ}{PT}$	6. Substitution
$7.  \frac{PS}{PS} + \frac{SR}{PS} = \frac{PT}{PT} + \frac{TQ}{PT}$	7. Commutative property of addition
$8. \frac{SR}{PS} = \frac{TQ}{PT}$	8. Subtraction property of equality

$\frac{PR}{PS} = \frac{PQ}{PT}$	$\frac{PS}{SR} = \frac{PT}{ST}$	∠P ≅ ∠P
AA Similarity	ASA Similarity	SSS Similarity
Reflexive property	Segment addition postulate	Corresponding sides of similar triangles are proportional.
Corresponding sides of similar triangles are congruent.	If two parallel lines are cut by a transversal, then alternate interior angles are congruent.	If two parallel lines are cut by a transversal, then alternate exterior angles are congruent.

Points Possible: 1

**Content Domain:** Similarity, Right Triangles, and Trigonometry

**Content Standard:** Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. (G.SRT.4)

# **Scoring Guidelines**

### Exemplar Response

Statements	Reasons
1. ST ∥ RQ	1. Given
2. ∠PST ≅ ∠R and ∠PTS ≅ ∠Q	If two parallel lines are cut by a transversal, then corresponding angles are congruent.
<ol><li>∆PQR ~ △PTS</li></ol>	3. AA Similarity
$4.  \frac{PR}{PS} = \frac{PQ}{PT}$	Corresponding sides of similar triangles are proportional.
5. $PR = PS + SR$ , $PQ = PT + TQ$	5. Segment addition postulate
6. $\frac{PS + SR}{PS} = \frac{PT + TQ}{PT}$	6. Substitution
7. $\frac{PS}{PS} + \frac{SR}{PS} = \frac{PT}{PT} + \frac{TQ}{PT}$	7. Commutative property of addition
$8. \frac{SR}{PS} = \frac{TQ}{PT}$	8. Subtraction property of equality

### Other Correct Responses

N/A

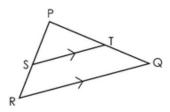
For this item, a full-credit response includes:

• A correct proof (1 point).

**Question 21** 

Sample Responses

Triangle PQR is shown, where  $\overline{ST}$  is parallel to  $\overline{RQ}$ .



Marta wants to prove that  $\frac{SR}{PS} = \frac{TQ}{PT}$ .

Place a statement or reason in each blank box to complete Marta's proof.

Statements	Reasons
1. <u>ST</u> ∥ <u>RQ</u>	1. Given
2. ∠PST ≅ ∠R and ∠PTS ≅ ∠Q	If two parallel lines are cut by a transversal, then corresponding angles are congruent.
3. △PQR ~ △PTS	3. AA Similarity
$4.  \frac{PR}{PS} = \frac{PQ}{PT}$	Corresponding sides of similar triangles are proportional.
5. $PR = PS + SR$ , $PQ = PT + TQ$	Segment addition postulate
6. $\frac{PS + SR}{PS} = \frac{PT + TQ}{PT}$	6. Substitution
$7.  \frac{PS}{PS} + \frac{SR}{PS} = \frac{PT}{PT} + \frac{TQ}{PT}$	7. Commutative property of addition
$8. \frac{SR}{PS} = \frac{TQ}{PT}$	8. Subtraction property of equality

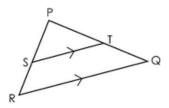
	$\frac{PS}{SR} = \frac{PT}{ST}$	∠P ≅ ∠P
	ASA Similarity	SSS Similarity
Reflexive property	Segment addition postulate	
Corresponding sides of similar triangles are congruent.	If two parallel lines are cut by a transversal, then alternate interior angles are congruent.	If two parallel lines are cut by a transversal, then alternate exterior angles are congruent.

#### **Notes on Scoring**

This response earns full credit (1 point) because it shows a correct proof of a theorem about a triangle where a line parallel to one side divides the other two proportionally.

In this situation, the existence of two pairs of congruent angles supports the statement about similarity of triangles PQR and PTS. Since a pair of parallel lines cut by two intersecting transversals form two triangles and two pairs of corresponding congruent angles, the triangles are similar by the Angle-Angle postulate. Having justified a similarity of the triangles, the next step is to select a statement showing a proportionality of sides,  $\frac{PR}{PS} = \frac{PQ}{PT}$ , along with correct reasoning (corresponding sides of similar figures are proportional).

Triangle PQR is shown, where ST is parallel to RQ.



Marta wants to prove that  $\frac{SR}{PS} = \frac{TQ}{PT}$ .

Place a statement or reason in each blank box to complete Marta's proof.

Statements	Reasons
1. <u>ST</u> ∥ <u>RQ</u>	1. Given
2. ∠PST ≅ ∠R and ∠PTS ≅ ∠Q	If two parallel lines are cut by a transversal, then corresponding angles are congruent.
3. △PQR ~ △PTS	3. AA Similarity
4.	4.
5. $PR = PS + SR, PQ = PT + TQ$	5. Segment addition postulate
$6. \frac{PS + SR}{PS} = \frac{PT + TQ}{PT}$	6. Substitution
$7.  \frac{PS}{PS} + \frac{SR}{PS} = \frac{PT}{PT} + \frac{TQ}{PT}$	7. Commutative property of addition
$8. \frac{SR}{PS} = \frac{TQ}{PT}$	8. Subtraction property of equality

$\frac{PR}{PS} = \frac{PQ}{PT}$	$\frac{PS}{SR} = \frac{PT}{ST}$	∠P ≅ ∠P
Corresponding sides of similar triangles are proportional.	ASA Similarity	SSS Similarity
Reflexive property	Segment addition postulate	
Corresponding sides of similar triangles are congruent.	If two parallel lines are cut by a transversal, then alternate interior angles are congruent.	If two parallel lines are cut by a transversal, then alternate exterior angles are congruent.

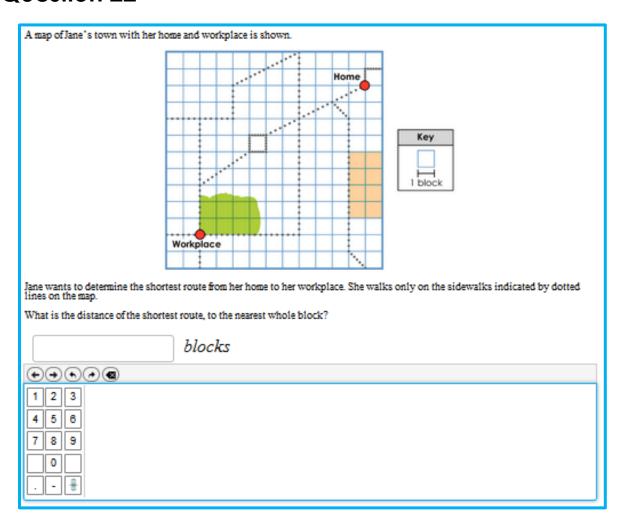
## **Notes on Scoring**

This response earns no credit (0 points) because it shows an incomplete proof (step 4 is missing) of a theorem about a triangle where a line parallel to one side divides the other two proportionally.

**Question 22** 

**Question and Scoring Guidelines** 

## **Question 22**



Points Possible: 1

Content Domain: Similarity, Right Triangles, and Trigonometry

**Content Standard:** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. (G.SRT.8)

# **Scoring Guidelines**

### Exemplar Response

• 15

### Other Correct Responses

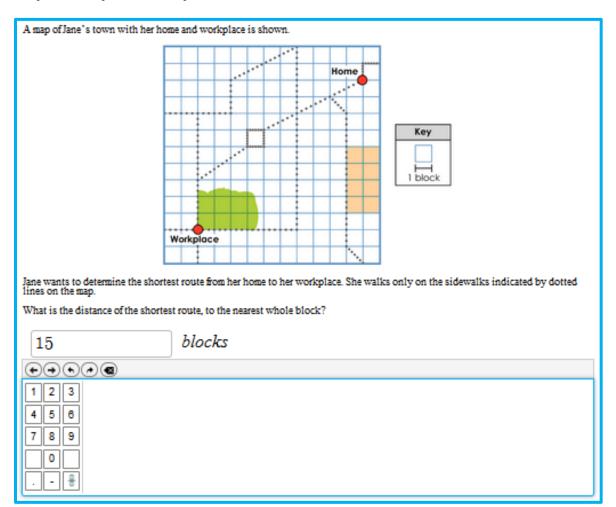
- Any value equivalent to 15 is accepted.
- Any value from 15.3 to 15.3138 is accepted.
- 16

For this item, a full-credit response includes:

• A correct distance (1 point).

**Question 22** 

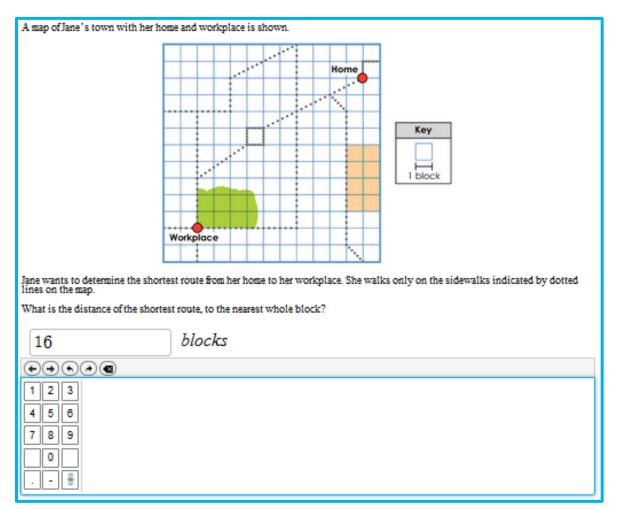
Sample Responses



#### **Notes on Scoring**

This response earns full credit (1 point) because it shows an acceptable length for the shortest route, rounded to the nearest whole block.

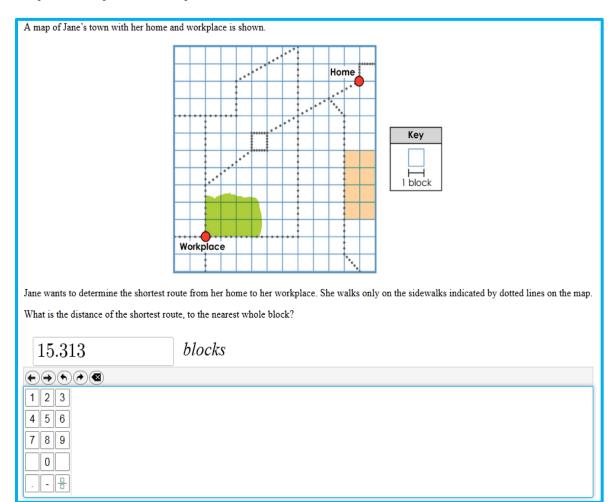
In this situation, determining the shortest distance from Home to Workplace involves choosing optimal walking routes and calculating their lengths. The shortest route consists of 5 portions. Using the Pythagorean Theorem, the length of the first portion, which is diagonal, is  $\sqrt{3^2+6^2}=\sqrt{45}$  blocks. The second and the third portions, along the sides of the square with length 1 block, are 1 block and 1 block. The fourth portion, which is another diagonal, is  $\sqrt{2^2+3^2}=\sqrt{13}$  blocks. The fifth portion, going vertically down, is 3 units. The total distance is  $\sqrt{45}+1+1+\sqrt{13}+3=15.313$  ..., which rounds to 15 whole blocks.



#### **Notes on Scoring**

This response earns full credit (1 point) because it shows an acceptable length for the shortest route, rounded to the nearest whole block.

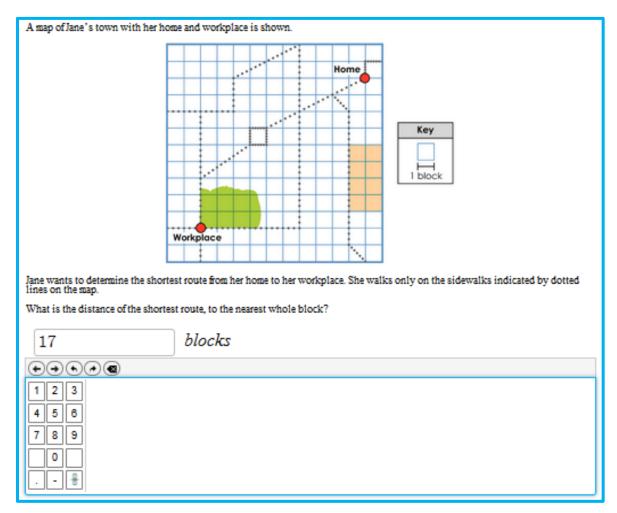
In this situation, determining the shortest distance from Home to Workplace involves choosing optimal walking routes and calculating their lengths. The shortest route consists of 5 portions. Using the Pythagorean Theorem, the length of the first portion, which is a diagonal, is  $\sqrt{3^2+6^2}=\sqrt{45}$  blocks. The second and the third portions, along the sides of the square with length 1 block, are 1 block and 1 block. The fourth portion, which is another diagonal, is  $\sqrt{2^2+3^2}=\sqrt{13}$  blocks. The fifth portion, going vertically down, is 3 units. The total distance is  $\sqrt{45}+1+1+\sqrt{13}+3=15.313\dots$  If each sq root is rounded up to the nearest whole number, the total distance is 16.



#### **Notes on Scoring**

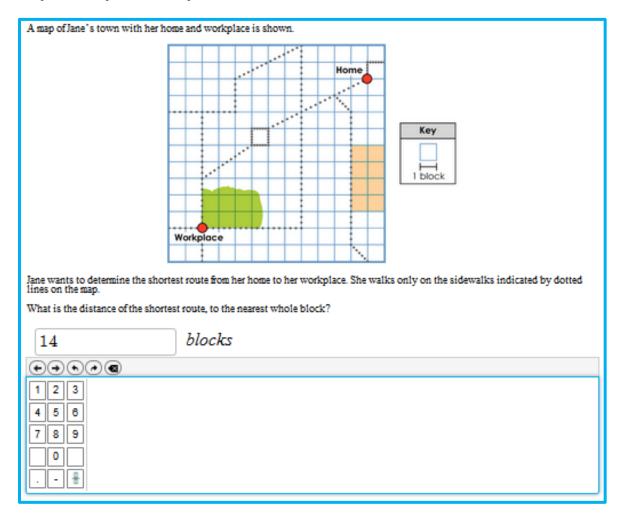
This response earns full credit (1 point) because it shows an acceptable length for the shortest route, not rounded to the nearest whole block.

In this situation, determining the shortest distance from Home to Workplace involves choosing optimal walking routes and calculating their lengths. The shortest route consists of 5 portions. Using the Pythagorean Theorem, the length of the first portion, which is a diagonal, is  $\sqrt{3^2+6^2}=\sqrt{45}$  blocks. The second and the third portions, along the sides of the square with length 1 block, are 1 block and 1 block. The fourth portion, which is another diagonal, is  $\sqrt{2^2+3^2}=\sqrt{13}$  blocks. The fifth portion, going vertically down, is 3 units. The total distance is  $\sqrt{45}+1+1+\sqrt{13}+3=15.313\dots$ 



### **Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect length of the shortest route, rounded to the nearest whole block.



### **Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect length of the shortest route, rounded to the nearest whole block.

**Question 23** 

**Question and Scoring Guidelines** 

## **Question 23**

An e	equa	atior	is shown, where $0 < x < 90$ and $0 < y < 90$ .
cos	xº)	= sir	n(y°)
Crea	ate	an e	expression for x in terms of y.
x	=		
•	•	(	
1	2	3	У
4	5	6	+ - • ÷
7	8	9	< ≤ = ≥ >
0	9	-	음 □ □ ()     √□ ♥□ π i
			sin cos tan arcsin arccos arctan

**Points Possible:** 1

Content Domain: Similarity, Right Triangles, and Trigonometry

**Content Standard:** Explain and use the relationship between the sine and cosine of complementary angles. (G.SRT.7)

# **Scoring Guidelines**

### Exemplar Response

• 90 − y

### Other Correct Responses

• Any equivalent expression.

For this item, a full-credit response includes:

• A correct expression (1 point).

**Question 23** 

Sample Responses

An equation is shown, where $0 < x < 90$ and $0 < y < 90$ .
$\cos(x^{\circ}) = \sin(y^{\circ})$
Create an expression for $x$ in terms of $y$ .
x = 90-y $1 2 3 y$
4 5 6 + - • ÷
7 8 9 < \leq = \geq >
0 Ε Ο Ο ()     √
sin cos tan arcsin arccos arctan

### **Notes on Scoring**

This response earns full credit (1 point) because it shows a correct expression for x in terms of y.

This situation recognizes a relationship between angle x and angle y. If the sine value of one angle equals the cosine value of another angle, or  $\sin x = \cos y$ , where 0 < y < 90, then angles are complementary. Angles are complementary if the sum of their measures is  $90^{\circ}$ , or x + y = 90. Therefore, x = 90 - y.

An equation	n is shown, where $0 < x < 90$ and $0 < y < 90$ .
$cos(x^o) = sir$	n(y°)
Create an e	expression for $x$ in terms of $y$ .
$x = \begin{bmatrix} - \\ - \\ \end{bmatrix}$	y+90 (♠) <b>(</b> ■)
1 2 3	У
4 5 6	+ - • ÷
7 8 9	< ≤ = ≥ >
0	
	sin cos tan arcsin arccos arctan
2	sin cos tan arcsin arccos arctan

## **Notes on Scoring**

This response earns full credit (1 point) because it is equivalent to the expression 90 - y.

This situation recognizes a relationship between angle x and angle y. If the sine value of one angle equals the cosine value of another angle, or  $\sin x = \cos y$ , where 0 < y < 90, then the angles are complementary. Angles are complementary if the sum of their measures is  $90^{\circ}$ , or x + y = 90. Therefore, x = 90 - y.

An eq	uatio	n is shown, where $0 < x < 90$ and $0 < y < 90$ .
cos(xº	) = sir	n(y°)
Create	an e	expression for $x$ in terms of $y$ .
x =	si	n(90-y)
$\bullet \bullet \bullet \bullet \otimes$		
1 2	3	у
4 5	6	+ - • ÷
7 8	9	< \leq = \geq >
0 .		
		sin cos tan arcsin arccos arctan

## **Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect expression for x in terms of y.

An e	equa	ation	n is shown, where $0 < x < 90$ and $0 < y < 90$ .
cos	(x°)	= sir	n(y°)
Crea	ate	an e	expression for $x$ in terms of $y$ .
x =	=	90	0+y
•	•	( <b>1</b> )	
1	2	3	У
4	5	6	+ - • ÷
7	8	9	< \leq = \geq >
0		-	
(B) (B)	60	50	sin cos tan arcsin arccos arctan

## **Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect expression for x in terms of y.

**Question 24** 

**Question and Scoring Guidelines** 

## **Question 24**

Two events, A and B, are independent.		
<ul> <li>P(A) = 0.3</li> <li>P(A and B) = 0.24</li> </ul>		
What is P(B)?		
P(B) =		
$\bullet \bullet \bullet \bullet \bullet \bullet$		
1 2 3		
4 5 6		
7 8 9		
· - B		

Points Possible: 1

Content Domain: Conditional Probability and the Rules of Probability

**Content Standard:** Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. (S.CP.2)

# **Scoring Guidelines**

#### Exemplar Response

• 0.8

## Other Correct Responses

• Any equivalent value.

For this item, a full-credit response includes:

• The correct probability (1 point).

**Question 24** 

Sample Responses

#### **Notes on Scoring**

This response earns full credit (1 point) because it shows the correct probability for the second of two independent events.

For two independent events A and B, the probability of them occurring together is the product of their probabilities.

If events A and B are independent, then  $P(A \text{ and B}) = P(A) \cdot P(B)$ . Therefore,  $0.24 = 0.3 \cdot P(B)$ , meaning P(B) = 0.8.

#### **Notes on Scoring**

This response earns full credit (1 point) because it shows the correct probability for the second of two independent events.

For two independent events A and B, the probability of them occurring together is the product of their probabilities. If events A and B are independent, then  $P(A \text{ and B}) = P(A) \cdot P(B)$ . Therefore,  $0.24 = 0.3 \cdot P(B)$ , meaning P(B) = 0.8 or 8/10.

### **Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect probability for the second of two independent events.

### **Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect probability for the second of two independent events.

**Question 25** 

**Question and Scoring Guidelines** 

#### **Question 25**

A total of 200 people attend a party, as shown in the table.

A person is selected at random to win a prize. The probability of selecting a female is 0.6. The probability of selecting a child, given that the person is female, is 0.25. The probability of selecting a male, given that the person is a child, is 0.4.

Complete the two-way table to show the number of adults, children, males, and females who attended the party.

	Adults	Children	Total
Male			80
Female			120
Total	150	50	200

**Points Possible:** 1

Content Domain: Conditional Probability and the Rules of Probability

**Content Standard:** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among Math, Science, and English. Estimate the probability that a randomly selected student from your school will favor Science given that the student is in tenth grade. Do the same for other subjects and compare the results. (S.CP.4)

# **Scoring Guidelines**

#### Exemplar Response

	Adults	Children	Total
Male	60	20	80
Female	90	30	120
Total	150	50	200

#### Other Correct Responses

• A table with equivalent values.

For this item, a full-credit response includes:

• A correct table (1 point).

**Question 25** 

Sample Responses

A total of 200 people attend a party, as shown in the table.

A person is selected at random to win a prize. The probability of selecting a female is 0.6. The probability of selecting a child, given that the person is female, is 0.25. The probability of selecting a male, given that the person is a child, is 0.4.

	Adults	Children	Total
Male	60	20	80
Female	90	30	120
Total	150	50	200

Complete the two-way table to show the number of adults, children, males, and females who attended the party.

#### **Notes on Scoring**

This response earns full credit (1 point) because it shows a completed table with correct values.

This situation requires a correct construction of a two-way frequency table of data. Since the probability of selecting a child, given it is a female, is 0.25, the number of female children is 120 • .25 or 30 (Row 2/Column 2). The number of female adults is 120 – 30 or 90 (Row 2/Column 1). Since the probability of selecting a male, given it is a child, is 0.4, the number of male children is 50 • 0.4 or 20 (Row 1/Column 2). The number of male adults is 80 – 20 or 60 (Row 1/Column 1).

A total of 200 people attend a party, as shown in the table.

A person is selected at random to win a prize. The probability of selecting a female is 0.6. The probability of selecting a child, given that the person is female, is 0.25. The probability of selecting a male, given that the person is a child, is 0.4.

Complete the two-way table to show the number of adults, children, males, and females who attended the party.

	Adults	Children	Total
Male	20	60	80
Female	30	90	120
Total	150	50	200

#### **Notes on Scoring**

This response earns no credit (0 points) because it shows a table with the correct values in the wrong cells.

A total of 200 people attend a party, as shown in the table.

A person is selected at random to win a prize. The probability of selecting a female is 0.6. The probability of selecting a child, given that the person is female, is 0.25. The probability of selecting a male, given that the person is a child, is 0.4.

Complete the two-way table to show the number of adults, children, males, and females who attended the party.

	Adults	Children	Total
Male	30	50	80
Female	120	0	120
Total	150	50	200

#### **Notes on Scoring**

This response earns no credit (0 points) because it shows a table with incorrect values.

A total of 200 people attend a party, as shown in the table.

A person is selected at random to win a prize. The probability of selecting a female is 0.6. The probability of selecting a child, given that the person is female, is 0.25. The probability of selecting a male, given that the person is a child, is 0.4.

	Adults	Children	Total
Male	80	0	80
Female	70	50	120
Total	150	50	200

Complete the two-way table to show the number of adults, children, males, and females who attended the party.

#### **Notes on Scoring**

This response earns no credit (0 points) because it shows a table with incorrect values.

**Question 26** 

**Question and Scoring Guidelines** 

#### **Question 26**

Sam is picking fruit from a basket that contains many different kinds of fruit.

Which set of events is independent?

- A Event 1: He picks a kiwi and eats it. Event 2: He picks an apple and eats it.
- Event 1: He picks a kiwi and eats it. Event 2: He picks a kiwi and puts it back.
- B Event 1: He picks an apple and eats it. Event 2: He picks an apple and eats it.
- Event 1: He picks a kiwi and puts it back. Event 2: He picks an apple and puts it back.

**Points Possible:** 1

Content Domain: Conditional Probability and the Rules of Probability

**Content Standard:** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer. (S.CP.5)

## **Scoring Guidelines**

<u>Rationale for Option A:</u> This is incorrect. The student may have thought that because the fruit were different, the two events must be independent, but missed that picking and eating a fruit without replacing it with the same fruit will affect the likelihood of picking a different fruit from the basket.

<u>Rationale for Option B:</u> This is incorrect. The student may have thought that, knowing that an apple was picked and eaten, P(E|E)=1 yields certainty for an apple to be picked and eaten.

<u>Rationale for Option C:</u> This is incorrect. The student may have switched the order of events and thought that because a kiwi was picked and put back, it did not affect the likelihood of picking another kiwi, which makes the two events independent.

<u>Rationale for Option D:</u> **Key** – The student correctly identified that picking a fruit from the basket and putting it back does not affect the likelihood of picking a different fruit and putting it back, which makes the two events independent.

#### Sample Response: 1 point

Sam is picking fruit from a basket that contains many different kinds of fruit.

Which set of events is independent?

Bevent 1: He picks a kiwi and eats it.
Event 2: He picks an apple and eats it.
Event 2: He picks an apple and eats it.
Event 1: He picks a kiwi and puts it back.

Event 1: He picks an apple and puts it back.
Event 2: He picks an apple and puts it back.
Event 2: He picks an apple and puts it back.

**Question 27** 

**Question and Scoring Guidelines** 

### **Question 27**

The thro	The probability of flipping a fair coin and heads landing face up is 0.5. The probability of rolling a fair number cube, with sides numbered 1 through 6, and an odd number landing face up is 0.5.				
Wha	at is	the	probability of flipping heads or rolling an odd number?		
_					
•	•	(			
1	2	3			
4	5	6			
7	8	9			
	0				
	-	0			

Points Possible: 1

Content Domain: Conditional Probability and the Rules of Probability

**Content Standard:** Apply the Addition Rule, P(A or B) = P(A) + P(B) - P(A and B), and interpret the answer in terms of the model. (S.CP.7)

## **Scoring Guidelines**

#### Exemplar Response

• 0.75

#### Other Correct Responses

• Any equivalent value.

For this item, a full-credit response includes:

• A correct value (1 point).

**Question 27** 

Sample Responses

The probability of flipping a fair coin and heads landing face up is 0.5. The probability of rolling a fair number cube, with sides numbered 1 through 6, and an odd number landing face up is 0.5.
What is the probability of flipping heads or rolling an odd number?
0.75
$\bullet \bullet \bullet \bullet \otimes$
4 5 6
7 8 9

#### **Notes on Scoring**

This response earns full credit (1point) because it shows the correct probability.

There are several strategies that can be used to solve problems with compound events. In this situation, the two compound events (i.e., events happening at the same time) are flipping heads (A) and rolling an odd number (B).

The Addition Rule, P(A or B) = P(A) + P(B) - P(A and B), can be used to calculate the probability of flipping heads or rolling an odd number. If the probability of flipping heads is 0.5 and the probability of rolling an odd number is 0.5, then the probability of flipping heads and rolling an odd number is 0.5 • 0.5 = 0.25, since these two events are independent. By substituting these values in the formula, the probability of flipping heads or rolling an odd number is P(A or B) = 0.5 + 0.5 - 0.25 = 0.75.

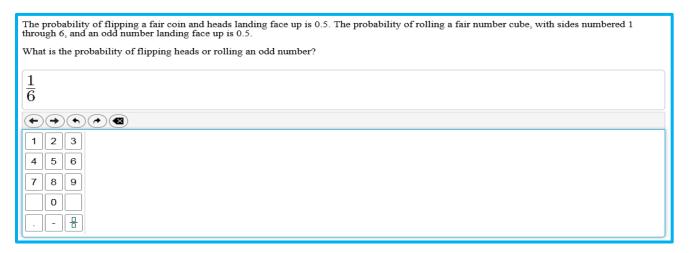
The probability of flipping a fair coin and heads landing face up is 0.5. The probability of rolling a fair number cube, with sides numbered 1 through 6, and an odd number landing face up is 0.5.
What is the probability of flipping heads or rolling an odd number?
$\left  rac{3}{4} \right $
<b>T</b>
4 5 6
7 8 9

#### **Notes on Scoring**

This response earns full credit (1point) because it shows the equivalent value of a correct probability.

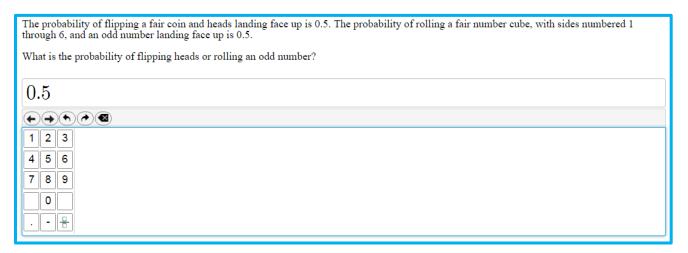
There are several strategies that can be used to solve problems with compound events. In this situation, the two compound events (i.e., events happening at the same time) are flipping heads (A) and rolling an odd number (B).

The Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), can be used to calculate the probability of flipping heads or rolling an odd number. If the probability of flipping heads is 0.5 and the probability of rolling an odd number is 0.5, then the probability of flipping heads and rolling an odd number is  $0.5 \cdot 0.5 = 0.25$ , since these two events are independent. By substituting these values in the formula, the probability of flipping heads or rolling an odd number is  $P(A \text{ or } B) = 0.5 + 0.5 - 0.25 = 0.75 \text{ or } \frac{3}{4}$ .



#### **Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect probability.



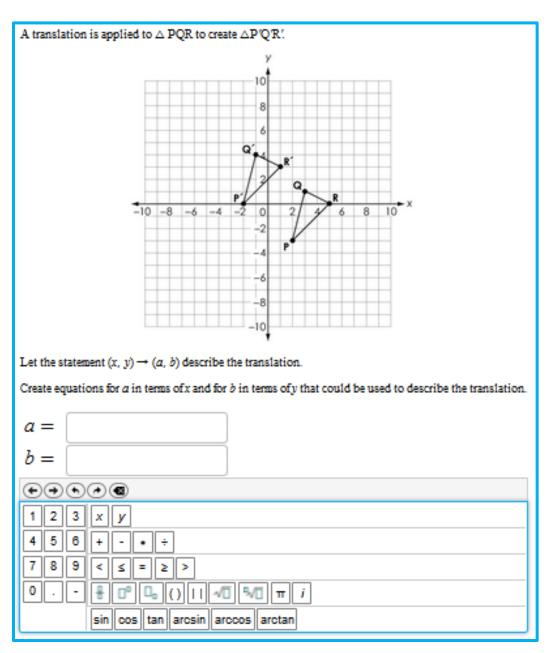
### **Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect probability.

**Question 28** 

**Question and Scoring Guidelines** 

### **Question 28**



Points Possible: 1

Content Domain: Congruence

**Content Standard:** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). (G.CO.2)

# **Scoring Guidelines**

#### Exemplar Response

• 
$$a = x - 4$$
  
 $b = y + 3$ 

### Other Correct Responses

• Any equivalent equation.

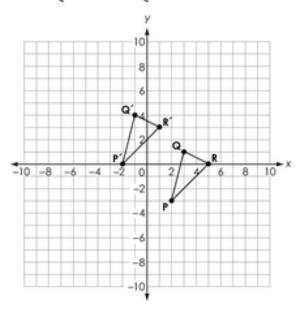
For this item, a full-credit response includes:

• A correct translation (1 point).

**Question 28** 

Sample Responses

A translation is applied to  $\triangle$  PQR to create  $\triangle$ P'Q'R'.



Let the statement  $(x, y) \rightarrow (a, b)$  describe the translation.

Create equations for  $\alpha$  in terms of x and for b in terms of y that could be used to describe the translation.

$$a = x - 4$$

$$b = y+3$$

_	_	_	_	_
(+)	( <b>+</b> )	(D)	(4)	(40)

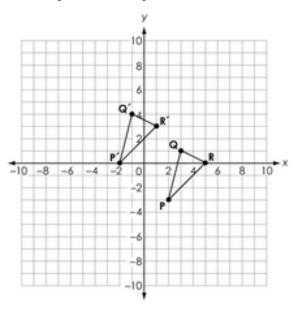
- 1 2 3 x y
- 4 5 6 + · ÷
- 7 8 9 < < = >
- 0 . 8 0° C₀ () II √0 %0 π i
  - sin cos tan arcsin arccos arctan

#### **Notes on Scoring**

This response earns full credit (1 point) because it shows a correct equation for a in terms of x, and for b in terms of y.

Under the translation of  $\triangle PQR$  to  $\triangle P'Q'R'$ , every point (x, y) of  $\triangle PQR$  corresponds to the point (a, b) of  $\triangle P'Q'R'$  or  $(x, y) \rightarrow (a, b)$ . Using one point as reference, it can be seen that each point of  $\triangle P'Q'R'$  is 4 units to the left and 3 units up from its corresponding point in  $\triangle PQR$ . Using coordinates, this translation can be described by adding -4 to the x-coordinate and adding positive 3 to the y-coordinate of the point (x, y) or  $(x, y) \rightarrow (x - 4, y + 3)$ . Therefore, a = x - 4 and b = y + 3.

A translation is applied to  $\triangle$  PQR to create  $\triangle$ P'Q'R'.



Let the statement  $(x, y) \rightarrow (a, b)$  describe the translation.

Create equations for  $\alpha$  in terms of x and for b in terms of y that could be used to describe the translation.

$$a = \boxed{-4+x}$$

$$b = \begin{vmatrix} 3+y \end{vmatrix}$$

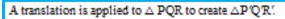
	_	_	_
( <b>+</b> )( <b>+</b> )	(A)		
	w	w	

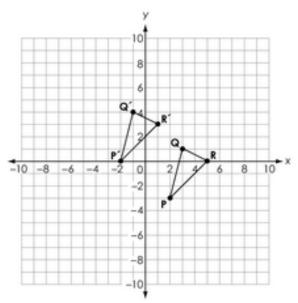
- 1 2 3 x y
- 4 5 6 + · ÷
- 7 8 9 < < = >
- 0 . 8 0° C () Π √0 5/0 π i
  - sin cos tan arcsin arccos arctan

#### **Notes on Scoring**

This response earns full credit (1 point) because it shows a correct equation for a in terms of x, and for b in terms of y.

Under the translation of  $\triangle PQR$  to  $\triangle P'Q'R'$ , every point (x, y) of  $\triangle PQR$  corresponds to the point (a, b) of  $\triangle P'Q'R'$  or  $(x, y) \rightarrow (a, b)$ . Using one point as reference, it can be seen that each point of  $\triangle P'Q'R'$  is 4 units to the left and 3 units up from its corresponding point in  $\triangle PQR$ . Using coordinates, this translation can be described by adding -4 to the x-coordinate and adding positive 3 to the y-coordinate of the point (x, y) or  $(x, y) \rightarrow (x - 4, y + 3)$ . Therefore, a = x - 4 and b = y + 3.





Let the statement  $(x, y) \rightarrow (a, b)$  describe the translation.

Create equations for  $\alpha$  in terms of x and for b in terms of y that could be used to describe the translation.

$$a = x+4$$

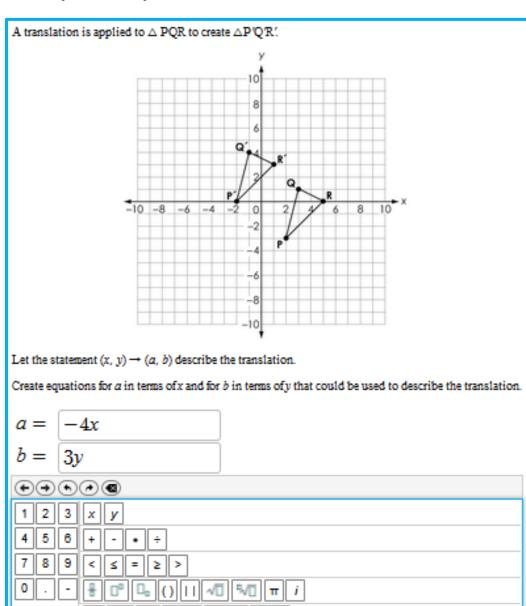
$$b = |y-3|$$

<b>(+)(+)</b>	(A)	(A)	<b>(49)</b>
	w	w	9

$\overline{}$				_
1	2	3	$  _{\mathbf{x}} $	l v
_				

**Notes on Scoring** 

This response earns no credit (0 points) because it shows an incorrect equation for a in terms of x and an incorrect equation for b in terms of y.



## **Notes on Scoring**

This response earns no credit (0 points) because it shows an incorrect equation for a in terms of x and an incorrect equation for b in terms of y.

sin cos tan arcsin arccos arctan

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